AN

ELEMENTARY

INTELLECTUAL ARITHMETIC,

CONTAINING NUMEROUS

ORIGINAL CONTRACTIONS

IN

MULTIPLICATION.

BY

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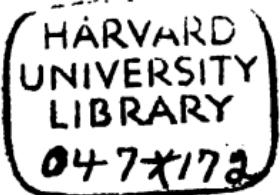
PREFACE.

In the preparation of this work, two objects have been constantly in view :— one was to prepare such gradual exercises for the pupils as are adapted to secure the most perfect intellectual training; the other was to develop and clearly unfold the principles that are usually found in written arithmetic,— so that, at the same time that the pupil is going through a thorough mental training, he is acquiring a clear knowledge of principles to be applied more fully afterwards.

While the work is especially designed to be studied before commencing written arithmetic, it is intended to be reviewed and studied by the pupil during his whole course; and, if so used, it will lead him more easily

to comprehend principles, and will greatly facilitate his application of them.

The contractions in multiplication, if made perfectly familiar both in their extent and application, will enable the pupil to abridge very materially nearly one half of the usual business processes in multiplication. These abridged processes can be applied with a very great saving in time and labor, in computing interest and other usual counting-room calculations.

The contractions, which are limited in their application, if they have but little practical value, will serve as excellent exercises for mental training.

CONTENTS.

ARITHMETIC.

ADDITION.

SECTION I.

ADDITION is the operation by which we collect two or more numbers of the same kind into one whole.

The result of Addition is called the *sum*.

1. John has 4 pens, and Charles has 2 pens; how many pens have they both together? SOLUTION. Four pens and 2 pens are 6 pens. ANS. 6 pens.

2. 6 pens and 5 pens are how many pens?

3. In the first class there are 6 girls, and in the second there are 3 girls; how many girls are there in both classes?

4. 7 girls and 4 girls are how many girls?

5. In the parlor there are 8 chairs, and in the dining-room there are 4 chairs; how many are there in both rooms?

6. 9 chairs and 5 chairs are how many chairs?

7. Joseph earned 9 cents, and his mother gave him 7 cents; how much money had he then?

8. 10 cents and 8 cents are how many cents?

9. William gave 10 cents for oranges, and 9 cents for a slate; what did he give for both?

10. A farmer paid 12 dollars for flour, and 8 dollars for corn; how many dollars did he pay?

11. There are 11 sheep in one pasture, and 8 in another; how many are there in both?

12. A lady paid 6 dollars for a bonnet, and 13 dollars for a shawl; how much did she pay for both?

13. William is 14 years old, and George is 7 ; what is the sum of their ages ?

14. 15 years and 8 years are how many years ?

15. If a cow is worth 16 dollars, and a pig is worth 8 dollars, what are both together worth ?

16. How many are 7 and 12 ?

17. How many are 14 and 6 ? 15 and 9 ? 13 and 8 ? 16 and 5 ? 18 and 7 ? 6 and 19 ?

18. How many are 8 and 16 ? 12 and 9 ? 4 and 15 ?

19. What two numbers may be added together to produce 12 ? ANS: 1 and 11; 2 and 10; 3 and 9; 4 and 8; 5 and 7; 6 and 6.

20. What two numbers added together will produce 14 ? 15 ? 16 ? 18 ?

21. What two numbers may be added together to produce 25 ? 28 ? 32 ? 45 ?

22. What is Addition ?

23. What is the result called ?

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SECTION II.

1. How many are 2 and 9 ? 9 and 2 ? 12 and 9 ? 19 and 2 ? 22 and 9 ? 29 and 2 ? 32 and 9 ? 39 and 2 ? 42 and 9 ? 49 and 2 ? 52 and 9 ? 59 and 2 ? 62 and 9 ? 69 and 2 ? 72 and 9 ? 79 and 2 ? 82 and 9 ? 89 and 2 ? 92 and 9 ? 99 and 2 ?

2. How many are 3 and 9 ? 9 and 3 ? 13 and 9 ? 19 and 3 ? 23 and 9 ? 29 and 3 ? 33 and 9 ? 39 and 3 ? 43 and 9 ? 49 and 3 ? 53 and 9 ? 59 and 3 ? 63 and 9 ? 69 and 3 ? 73 and 9 ? 79 and 3 ? 83 and 9 ? 89 and 3 ? 93 and 9 ? 99 and 3 ?

3. How many are 4 and 9 ? 9 and 4 ? 14 and 9 ? 19 and 4 ? 24 and 9 ? 29 and 4 ? 34 and 9 ? 39 and 4 ? 44 and 9 ? 49 and 4 ? 54 and 9 ? 59 and 4 ? 64 and 9 ? 69 and 4 ? 74 and 9 ? 79 and 4 ? 84 and 9 ? 89 and 4 ? 94 and 9 ? 99 and 4 ?

4. How many are 5 and 9? 9 and 5? 15 and 9?
19 and 5? 25 and 9? 29 and 5? 35 and 9? 39 and
5? 45 and 9? 49 and 5? 55 and 9? 59 and 5? 65
and 9? 69 and 5? 75 and 9? 79 and 5? 85 and 9?
89 and 5? 95 and 9? 99 and 5?

5. How many are 6 and 9? 9 and 6? 16 and 9? 19
and 6? 26 and 9? 29 and 6? 36 and 9? 39 and 6?
46 and 9? 49 and 6? 56 and 9? 59 and 6? 66 and
9? 69 and 6? 76 and 9? 79 and 6? 86 and 9? 89
and 6? 96 and 9? 99 and 6?

6. How many are 7 and 9? 9 and 7? 17 and 9?
19 and 7? 27 and 9? 29 and 7? 37 and 9? 39 and
7? 47 and 9? 49 and 7? 57 and 9? 59 and 7? 67
and 9? 69 and 7? 77 and 9? 79 and 7? 87 and 9? 89
and 7? 97 and 9? 99 and 7?

7. How many are 8 and 9? 9 and 8? 18 and 9?
19 and 8? 28 and 9? 29 and 8? 38 and 9? 39 and
8? 48 and 9? 49 and 8? 58 and 9? 59 and 8? 68
and 9? 69 and 8? 78 and 9? 79 and 8? 88 and 9?
89 and 8? 98 and 9? 99 and 8?

8. How many are 9 and 9? 19 and 9? 29 and 9?
3 and 7? 13 and 7? 53 and 7? 47 and 3? 6 and 8?
16 and 8? 46 and 8? 48 and 6? 58 and 6? 68 and
6? 66 and 8? 76 and 8? 86 and 8? 88 and 6? 98
and 6?

9. How many are 3 and 13? 4 and 18? 5 and 17?
6 and 16? 26 and 7? 36 and 7? 46 and 7? 47 and
6? 57 and 6? 56 and 7? 76 and 7? 77 and 6? 87
and 6? 86 and 7? 96 and 7? 97 and 6? 97 and 7?
107 and 7?

10. How many are 4 and 1? 4 and 41? 4 and 61?
4 and 3? 4 and 28? 4 and 53? 4 and 63? 4 and 83?
14 and 3? 24 and 3? 34 and 3? 94 and 3? 93 and
4? 5 and 5? 5 and 15? 5 and 25? 5 and 45? 5 and
55? 5 and 6? 5 and 16? 5 and 26? 5 and 36? 86
and 5? 35 and 6?

SECTION III.

1. MARIA had 4 cents, Lucy gave her 5 cents, and John gave her 6; how many had she then? SOLUTION. 4 cents and 5 cents are 9 cents, and 6 cents are 15 cents.   ANS. 15 cents.

2. George paid 8 cents for a book, 7 cents for pens, and 6 cents for pencils; how much did he pay for all?

3. 9 and 8 and 10 are how many?

4. A farmer paid 8 dollars for butter, 6 dollars for flour, and 4 dollars for sugar; what did they all cost?

5. William bought a hat for 5 dollars, a pair of boots for 6 dollars, and a vest for 7 dollars; what did he pay for the whole?

6. 11 and 10 and 9 are how many?

7. If a farmer collected 6 bushels of apples from one tree, 4 bushels from another, and 11 bushels from a third, how many bushels did he gather from all?

8. 10 and 7 and 3 are how many?

9. In an orchard there are four rows of trees: in the first row there are 12 trees, in the second 8, in the third 6, in the fourth 4.   How many trees are there in the orchard?

10. Four men gave some money in charity: the first gave 11 dollars, the second 8, the third 7, and the fourth 5.   How much did they all contribute?

11. 9 and 10 and 4 and 8 are how many?

12. A man has money in four purses; in one he has 15 dollars, in another 9, in another 7, and in the fourth 8; how much money has he in all?

13. 15 and 9 and 9 and 6 are how many?

14. A boy gave 5 cents for some marbles, he paid 8 cents more for a ball than for the marbles, and for a top he paid 10 cents; how much did they all cost?

15. How many are 5 and 8 and 10?

16. In the first class there are 4 little boys, in the

second there are 3 more than in the first, and in the third there are two more than in the second; what is the number of boys in the three classes?

17. 4 and 7 and 9 are how many?

18. The fourth class use a Reader that costs 15 cents, a Speller that costs 12 cents, a Writing-book that costs 8 cents, and an Arithmetic that costs 10 cents; what does each scholar in the class pay for his books?

19. 15 and 12 and 9 and 10 are how many?

20. On the fourth of July, Henry spent 4 cents for an orange; for an ice-cream he gave 2 cents more than for the orange, for a pine-apple 6 cents more than for the ice-cream, and he paid 13 cents for riding in the cars; how much money did he spend?

21. 4 and 6 and 11 and 13 are how many?

22. A trader has 62 dollars in bank-notes, 9 dollars in gold, and 5 dollars in silver; how much money has he in all?

23. 63 and 9 and 5 are how many?

24. In an orchard there are 45 apple trees, 8 pear trees, 6 cherry trees, and 7 quince trees; how many trees are there in the orchard?

25. 53 and 7 and 6 and 5 are how many?

26. A farmer has three cows, valued at 75 dollars, a calf worth 8 dollars, three sheep worth 9 dollars, and a pig worth 7 dollars; how much are they all worth?

27. 49 and 9 and 9 and 9 are how many?

28. Henry had 39 cents; his father gave him 10 cents, his mother gave him 9 cents, and he found 5 cents; how much money had he then?

29. 57 and 8 and 8 and 8 are how many?

30. A gentleman paid 23 dollars for his coat, 9 dollars for his pantaloons, 7 dollars for his vest, 5 dollars for his hat, and 6 dollars for his boots; what did he pay for them all?

31. 28 and 7 and 6 and 5 and 4 are how many?

32. 46 and 7 and 7 and 7 and 9 are how many?

SECTION IV.

THE learner should be enabled to state promptly the tens and units of which any number, larger than ten, is composed. Thus, the number twelve (12) is composed of one ten and two units.

State the tens and units of which the following numbers are composed:

14	37	65	86
16	39	69	91
19	41	71	93
23	43	79	95
29	46	81	100
31	57	83	110
34	62	85	120

By taking the tens and the units separately, children may be taught to add with great rapidity.

How many are 37 and 45? SOLUTION. 3 tens and 4 tens are 7 tens; 7 units and 5 units are 1 ten and 2 units; 7 tens and 1 ten and 2 units are 8 tens and 2 units, or 82. ANS. 82.

When the operation is thoroughly understood, the answer may be given without explanation.

The sign + (plus), which signifies *added to*, or *and*, is often used in addition. Thus: 8 + 6 is the same as 8 and 6.

1. How many are 50 + 12? 60 + 14? 70 + 18?
2. How many are 60 + 14? 70 + 15? 80 + 19?
3. How many are 50 + 16? 60 + 15? 70 + 17?
4. How many are 50 + 18? 60 + 16? 70 + 16?
5. How many are 60 + 12? 70 + 14? 80 + 18?
6. How many are 60 + 18? 40 + 13? 20 + 11?
7. How many are 70 + 19? 80 + 18? 90 + 17?
8. How many are 13 + 17? 14 + 18? 15 + 19? 16 + 13? 17 + 14? 19 + 14? 16 + 17?

16 + 19? 17 + 18? 18 + 19? 19 + 19? 16 + 16? 17 + 17? 18 + 18?

9. How many are 16 + 21? 17 + 23? 18 + 24? 19 + 25? 18 + 28? 17 + 29? 16 + 27?

10. How many are 21 + 22? 23 + 24? 25 + 27? 26 + 23? 27 + 33? 32 + 34? 35 + 42? 37 + 34?

11. How many are 43 + 34? 43 + 44? 45 + 46? 54 + 43? 55 + 56? 57 + 58? 59 + 57?

12. How many are 63 + 65? 64 + 67? 66 + 67? 68 + 42? 67 + 57? 64 + 59?

13. How many are 72 + 74? 77 + 84? 73 + 77? 84 + 85? 88 + 64? 92 + 84? 87 + 56?

14. How many are 110 + 54? 122 + 65? 138 + 75? 149 + 86? 187 + 96?

15. How many are 10 + 20 + 30 + 40? 34 + 40 + 50 + 60?

16. How many are 20 + 30 + 40 + 50? 40 + 60 + 70 + 80?

17. How many are 60 + 70 + 60 + 30? 20 + 30 + 50 + 90?

18. How many are 20 + 30 + 5 + 4? 30 + 40 + 7 + 6?

19. How many are 30 + 40 + 6 + 8? 50 + 70 + 5 + 8?

20. How many are 25 + 34? 36 + 48? 87 + 46? 55 + 78?

21. How many are 40 + 50 + 7 + 8? 30 + 80 + 8 + 6?

22. How many are 60 + 30 + 9 + 6? 40 + 90 + 6 + 7?

23. How many are 47 + 58? 69 + 36? 38 + 86? 46 + 79?

24. How many are 50 + 40 + 9 + 8? 20 + 80 + 6 + 9?

25. How many are 70 + 80 + 6 + 7? 40 + 60 + 7 + 9?

MULTIPLICATION.

SECTION V.

WHEN *equal* numbers are to be added, we proceed by a shorter method, called Multiplication. Thus: 4 and 4 and 4 are 12, is expressed, by Multiplication, 3 times 4 are 12.

Multiplication is the process of finding the sum of any number, when taken as many times as there are units in another number.

OBS. This definition of Multiplication is applicable only to whole numbers.

The number produced by multiplication is called the *product.* The number multiplied is called the *multiplicand.* The number multiplied by is called the *multiplier.* The multiplier and the multiplicand are called *factors* of the product. One of the factors must always be regarded as an abstract number, showing the times the other is taken.

1. How many are 2 times 1? 2 times 2? 2 times 3? 2 times 4? 2 times 5? 2 times 6? 2 times 7? 2 times 8? 2 times 9? 2 times 10? 2 times 11? 2 times 12?

2. How many are 3 times 1? 3 times 2? 3 times 3? 3 times 4? 3 times 5? 3 times 6? 3 times 7? 3 times 8? 3 times 9? 3 times 10? 3 times 11? 3 times 12?

3. How many are 4 times 1? 4 times 2? 4 times 3? 4 times 4? 4 times 5? 4 times 6? 4 times 7? 4 times 8? 4 times 9? 4 times 10? 4 times 11? 4 times 12?

4. How many are 5 times 1? 5 times 2? 5 times 3? 5 times 4? 5 times 5? 5 times 6? 5 times 7? 5 times 8? 5 times 9? 5 times 10? 5 times 11? 5 times 12?

5. How many are 6 times 1? 6 times 2? 6 times 3? 6 times 4? 6 times 5? 6 times 6? 6 times 7? 6 times 8? 6 times 9? 6 times 10? 6 times 11? 6 times 12?

6. How many are 7 times 1? 7 times 2? 7 times 3? 7 times 4? 7 times 5? 7 times 6? 7 times 7? 7 times 8? 7 times 9? 7 times 10? 7 times 11? 7 times 12?

7. How many are 8 times 1? 8 times 2? 8 times 3? 8 times 4? 8 times 5? 8 times 6? 8 times 7? 8 times 8? 8 times 9? 8 times 10? 8 times 11? 8 times 12?

8. How many are 9 times 1? 9 times 2? 9 times 3? 9 times 4? 9 times 5? 9 times 6? 9 times 7? 9 times 8? 9 times 9? 9 times 10? 9 times 11? 9 times 12?

9. How many are 10 times 1? 10 times 2? 10 times 3? 10 times 4? 10 times 5? 10 times 6? 10 times 7? 10 times 8? 10 times 9? 10 times 10? 10 times 11? 10 times 12?

10. How many are 11 times 1? 11 times 2? 11 times 3? 11 times 4? 11 times 5? 11 times 6? 11 times 7? 11 times 8? 11 times 9? 11 times 10? 11 times 11? 11 times 12?

11. How many are 12 times 1? 12 times 2? 12 times 3? 12 times 4? 12 times 5? 12 times 6? 12 times 7? 12 times 8? 12 times 9? 12 times 10? 12 times 11? 12 times 12?

12. How many are 2 times 7? 3 times 11? 4 times 9? 5 times 8? 6 times 5? 7 times 4? 8 times 6? 9 times 12? 10 times 10? 11 times 8? 12 times 9? 7 times 12?

13. What is Multiplication?
14. What is the product?
15. What is the multiplicand?
16. What is the multiplier?
17. What are factors?

SECTION VI.

1. JAMES bought 9 oranges at 3 cents apiece; how much did he give for them all? SOLUTION. If one orange cost 3 cents, 9 oranges will cost 9 times 3 cents, which are 27 cents. ANS. 27 cents.

2. $3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$ are how many? How many times is 3 taken to make 27?

OBS. The pupil must be made to understand thoroughly the analogy between Multiplication and Addition.

3. There are 6 shillings in one dollar; how many shillings are there in 4 dollars?

4. 6 and 6 and 6 and 6 are how many?

5. If a horse travel 9 miles in one hour, how far will he travel in 5 hours?

6. $9 + 9 + 9 + 9 + 9$ are how many? How many times is 9 taken?

7. In one yard there are 3 feet; how many feet are there in 7 yards? in 9 yards? 11 yards?

8. Seven days make one week; how many days are there in 6 weeks? 8 weeks? 10 weeks? 12 weeks?

9. What cost 9 yards of cloth, at 8 dollars a yard?

10. If 9 men can do a piece of work in 5 days, how many men would it take to do it in one day?

11. When wood is worth 6 dollars a cord, what is the value of 7 cords?

12. A laborer works 10 hours each day; how many hours does he work in 8 days?

13. What cost 7 knives, at 8 cents apiece?

14. 8 and 8 and 8 and 8 and 8 and 8 and 8 are how many?

15. Owen bought 9 books, at 9 cents each; how many cents did he pay for the whole?

16. What will be the cost of 8 pounds of beef, at 7 cents a pound?

17. In 1 pint there are 4 gills; how many gills in 6 pints? in 8 pints? 9 pints? 10 pints?

18. In 1 gallon there are 8 pints; how many pints in 7 gallons? in 5 gallons? in 8 gallons? in 10 gallons?

19. At 3 dollars a gallon, what will 12 gallons of wine cost?

20. If a ship sail 11 miles in an hour, how far would she sail in 7 hours? in 4 hours? in 8 hours? 10 hours?

21. A cannon-ball, when first discharged, flies at the rate of 9 miles a minute; how far would it fly, at that rate, in 4 minutes? in 7 minutes?

Obs. The sign of Multiplication is a cross, $\times$, and is read, multiplied by, or times.

22. How many are 6 times 5? 4 $\times$ 8? 5 $\times$ 6? 7 $+$ 4? 6 $\times$ 7?

23. 4 times 7 are how many? 5 times 8? 7 times 5? 7 times 6? 8 times 4?

24. How many are 7 times 7? 8 $\times$ 6? 9 $\times$ 5? 10 $\times$ 3? 8 $\times$ 8?

25. 9 times 6 are how many? 10 times 9? 11 times 8? 12 times 6? 11 times 12?

26. If the interest of 1 dollar is 6 cents for one year, what is the interest of 8 dollars for the same time? of 9 dollars? 4 dollars? 12 dollars?

27. If the interest of a dollar is 5 mills for one month, what is the interest of one dollar for 6 months? 8 months? 10 months?

28. If a man earn 12 dollars in each month, how much money will he earn in 6 months? 8 months? 11 months?

29. A man travels 9 hours a day, and 7 miles each hour; how far does he travel in a day?

30. 9 times 7 are how many sevens? are how many?

31. 4 times 4 tens are how many tens? are how many?

2

32. 7 times 6 tens are how many tens? are how many?

33. 8 times 5 tens are how many tens? are how many?

34. 3 times 9 tens are how many tens? are how many?

35. 4 times 8 tens are how many tens? are how many?

ADDITION COMBINED WITH MULTIPLICATION.

SECTION VII.

1. 2 tens and 3 tens are how many tens? are how many?

2. 4 tens and 6 tens are how many tens? are how many?

3. 8 tens and 6 tens are how many tens? are how many?

4. 7 tens and 4 tens and 5 tens are how many tens? are how many?

5. 8 tens and 6 tens and 7 tens are how many tens? are how many?

6. John has 7 books; Mary has 8 times as many, and Lucy has 3 times as many, as John; how many books has each girl? how many times 7 books, and what number of books, have all together?

7. 2 sevens and 4 sevens and 5 sevens are how many sevens? are how many?

8. Albert has 5 pens; Robert has 5 times as many, and Charles 6 times as many, as Albert; how many times 5 pens, and how many pens, do they all possess?

9. 3 fives and 6 fives and 6 fives are how many fives ? are how many ?

10. A man purchases 6 times as many sheep, and 3 times as many cows, as he does horses, and the number of horses is 4; how many sheep and cows does he buy ? how many times 4 animals, and how many animals, in all ?

11. 6 times 4, and 3 times 4, and 2 times 4, are how many times 4 ? are how many ?

12. A laborer has three pieces of work to do; the first he can perform in 8 days, the second in 3 times the time required for the first, and the third in twice the time required for the first and second together; how many times 8 days, and what number of days, will it take to do the whole ?

13. Once 8 and 3 times 8 and twice 4 times 8 are how many times 8 ? are how many ?

14. In an orchard there are 9 pear trees, 3 times as many cherry trees, and 5 times as many apple trees ; how many cherry trees and how many apple trees are there in the orchard ? how many times 9 trees, and how many trees, in all ?

15. 3 times 2 nines, + 2 times 2 nines are how many nines ? are how many ?

16. Three men purchase a pleasure-boat; the first pays 11 dollars, the second 6 times as much, and the third 4 times as much, as the first; how many times 11 dollars does the boat cost ? how much does the second and the third man pay ? what is the cost of the boat ?

17. 3 times 11 and 4 times 11 and 5 times 11 are how many times 11 ? are how many ?

18. In a factory there are 4 times as many men and 3 times as many women as children, and the number of children is 12; how many times 12 persons, and what number of persons, are in the factory ?

19. 2 times 12, + 3 times 12, + 3 times 12, + 4 times 12 are how many times 12 ? are how many ?

MULTIPLICATION.

SECTION VIII.

In performing the examples in this section, the pupil may be taught to multiply the tens and the units separately.

How many are 8 times 18? SOLUTION. 8 times 1 ten are 8 tens; 8 times 8 units are 64 units, or 6 tens and 4 units; 8 tens ─┼─ 6 tens and 4 units are 14 tens and 4 units. ANS. 144.

1. How many are 2 times 13? 2 times 14? 2 times 15? 2 times 16? 2 times 17? 2 times 18? 2 times 19? 2 times 20?

2. How many are 3 times 13? 3 times 14? 3 times 15? 3 times 16? 3 times 17? 3 times 18? 3 times 19? 3 times 20?

3. How many are 4 times 13? 4 times 14? 4 times 15? 4 times 16? 4 times 17? 4 times 18? 4 times 19? 4 times 20?

4. How many are 5 times 13? 5 times 14? 5 times 15? 5 times 16? 5 times 17? 5 times 18? 5 times 19? 5 times 20?

5. How many are 6 times 13? 6 times 14? 6 times 15? 6 times 16? 6 times 17? 6 times 18? 6 times 19? 6 times 20?

6. How many are 7 times 13? 7 times 14? 7 times 15? 7 times 16? 7 times 17? 7 times 18? 7 times 19? 7 times 20?

7. How many are 8 times 13? 8 times 14? 8 times 15? 8 times 16? 8 times 17? 8 times 18? 8 times 19? 8 times 20?

8. How many are 9 times 13? 9 times 14? 9 times 15? 9 times 16? 9 times 17? 9 times 18? 9 times 19? 9 times 20?

SUBTRACTION.

SECTION IX.

SUBTRACTION is the process of finding the difference between two numbers of the same kind.

OBS. It is the opposite of Addition.

The larger number is called the *minuend;* the smaller, the *subtrahend;* the answer is called the *remainder.*

A short, horizontal line, — (read *less*), is the sign used to express Subtraction.

1. 4 and 3 are how many? take 3 from 7, and what remains?

2. 8 and 5 are how many? take 5 from 13, and what remains?

3. 6 + 7 are how many? Subtract 6 from 13, and what remains? take 7 from 13, and what remains?

4. How many are 7 and 9? Subtract 9 from 16, and what remains?

5. 9 + how many are 18?

6. 7 + how many are 16?

7. 12 + how many are 18?

8. 18 — how many are 9?

9. 25 — how many are 17?

10. 23 — how many are 11?

11. 11 + how many are 23?

12. 17 + how many are 25?

13. A man paid 17 dollars for some sheep and a calf; for the sheep he paid 12 dollars; what was the price of the calf? SOLUTION. If he paid 17 dollars for the sheep and calf together, and 12 dollars for the sheep alone, the calf must have cost the difference between 17 dollars and 12 dollars, which is 5 dollars. ANS. 5 dollars.

14. If a barrel of flour be worth 7 dollars, and a bag of coffee 13 dollars, what is the difference in their value ?

15. 13 + how many are 20 ?

16. 20 — how many are 7 ?

17. In the first class there are 11 boys, in the second 15 ; how many more boys are there in the second class than in the first ?

18. There were 16 barrels upon the wharf, but the express man took away 8 of them ; how many were left upon the wharf ?

19. 16 + how many are 25 ?

20. 25 — how many are 16 ?

21. In my garden there are rose-bushes bearing red roses and white roses, of each an equal number ; I gathered 13 white roses and 20 red ones ; by how many do the white roses remaining in the garden exceed the red ?

22. 14 + how many are 25 ?

23. 25 — how many are 14 ?

24. Frank gathered 24 russet apples from one tree, and 12 baldwin apples from another ; how many had he of one kind more than of the other ?

25. 15 + how many are 27 ?

26. 27 — how many are 15 ?

27. 30 — how many are 20 ?

28. 19 — 5 are how many ? 5 and how many are 19 ?

29. 9 tens less 2 tens are how many tens ? are how many units ?

30. 19 tens less 8 tens are how many tens ? are how many units ?

31. 30 tens — how many tens are 12 tens ? are how many units ?

32. What is Subtraction ?

33. What name is given to the larger number ? to the smaller number ?

34. What name is applied to the answer ?

SECTION X.

1. How many are 9 less 2 ? 9 less 7 ? 19 less 2 ? 19 less 7 ? 29 less 2 ? 29 less 7 ? 39 less 2 ? 39 less 7 ? 49 less 2 ? 49 less 7 ? 59 less 2 ? 59 less 7 ? 69 less 2 ? 69 less 7 ? 79 less 2 ? 79 less 7 ? 89 less 2 ? 89 less 7 ? 99 less 2 ? 99 less 7 ? 109 less 2 ? 109 less 7 ?

2. How many are 9 less 3 ? 9 less 6 ? 19 less 3 ? 19 less 6 ? 29 less 3 ? 29 less 6 ? 39 less 3 ? 39 less 6 ? 49 less 3 ? 49 less 6 ? 59 less 3 ? 59 less 6 ? 69 less 3 ? 69 less 6 ? 79 less 3 ? 79 less 6 ? 89 less 3 ? 89 less 6 ? 99 less 3 ? 99 less 6 ? 109 less 3 ? 109 less 6 ?

3. How many are 9 less 4 ? 9 less 5 ? 19 less 4 ? 19 less 5 ? 29 less 4 ? 29 less 5 ? 39 less 4 ? 39 less 5 ? 49 less 4 ? 49 less 5 ? 59 less 4 ? 59 less 5 ? 69 less 4 ? 69 less 5 ? 79 less 4 ? 79 less 5 ? 89 less 4 ? 89 less 5 ? 99 less 4 ? 99 less 5 ? 109 less 4 ? 109 less 5 ?

4. How many are 8 less 5 ? 8 less 3 ? 18 less 5 ? 18 less 3 ? 28 less 5 ? 28 less 3 ? 38 less 5 ? 38 less 3 ? 48 less 5 ? 48 less 3 ? 58 less 5 ? 58 less 3 ? 68 less 5 ? 68 less 3 ? 78 less 5 ? 78 less 3 ? 88 less 5 ? 88 less 3 ? 98 less 5 ? 98 less 3 ? 108 less 5 ? 108 less 3 ?

5. How many are 7 less 4 ? 7 less 3 ? 17 less 4 ? 17 less 3 ? 27 less 4 ? 27 less 3 ? 37 less 4 ? 37 less 3 ? 47 less 4 ? 47 less 3 ? 57 less 4 ? 57 less 3 ? 67 less 4 ? 67 less 3 ? 77 less 4 ? 77 less 3 ? 87 less 4 ? 87 less 3 ? 97 less 4 ? 97 less 3 ? 107 less 4 ? 107 less 3 ?

6. How many are 8 less 7 ? 18 less 7 ? 58 less 7 ? 68 less 7 ? 88 less 7 ? 98 less 7 ? 108 less 7 ? 8 less 6 ? 8 less 2 ? 18 less 6 ? 18 less 2 ? 28 less 6 ? 48 less 6 ? 68 less 6 ? 78 less 6 ? 98 less 6 ? 108 less 6 ? 28 less 2 ? 68 less 2 ? 78 less 2 ? 88 less 2 ? 108 less 2 ?

7. How many are 7 less 5 ? 17 less 5 ? 27 less 5 ?
37 less 5 ? 47 less 5 ? 57 less 5 ? 67 less 5 ? 77 less
5 ? 87 less 5 ? 97 less 5 ? 7 less 6 ? 17 less 6 ? 27
less 6 ? 37 less 6 ? 47 less 6 ? 57 less 6 ? 67 less 6 ?
77 less 6 ? 87 less 6 ? 97 less 6 ? 107 less 6 ?

ADDITION COMBINED WITH SUB-TRACTION.

SECTION XI.

1. A MAN has 7 horses in his stable and 2 in the
pasture; how many has he in both places ? how many
more in the stable than in the pasture ?

2. 6 + 7 + how many are 20 ?

3. A lady paid 20 dollars for a muff and 9 dollars
for a tippet; what did she pay for both ? what is the
difference in their value ?

4. 9 + 2 + how many are 20 ?

5. Charles spent 5 cents for marbles, 8 cents for a
ball, and 12 cents for a top; how much did he pay for
the toys ? how much more for the marbles and top than
for the ball ?

6. 8 + how many are 12 + 5 ?

7. Lucy had 12 cents; her father gave her 10 more,
her mother 5, and she net a purse which she sold for 25
cents; how much money had she remaining, after pur-
chasing a book for 37 cents ?

8. 6 + 9 + 8 + how many are 30 ?

9. A grocer sells a firkin of butter for 13 dollars, a
box of raisins for 2 dollars, a can of oil for 6 dollars, and
a bag of coffee for 14 dollars; he receives in payment a
fifty-dollar bill; how much does he return to the pur-
chaser ?

10. Henry's pony cost 48 dollars, the saddle 15 dollars, and the bridle 4 dollars; he sold them for 75 dollars; did he gain or lose? how much?

11. Five gentlemen agree to contribute 100 dollars toward the Washington Monument; the first gives 16, the second 18, the third 34, and the fourth 21 dollars; how many dollars does the fifth man contribute, to complete the subscription?

12. What three numbers, taken together, will amount to 31?

13. What three numbers, taken together, will amount to 51?

Obs. The pupil may add any two numbers, whose sum is less than the required number, and subtract their sum from the number.

It will prove a good exercise, for several pupils to perform the same example, each one giving different numbers.

14. What four numbers, taken together, will amount to 17? 25? 33? 38?

15. What five numbers, taken together, will amount to 17? 25? 33? 38?

16. The expense of sustaining a village school, for four months, is 124 dollars, and it is paid by eight gentlemen; the first pays 12 dollars, the second 13, the third 14, the fourth 15, the fifth 16, the sixth 17, the seventh 18, and the eighth the remainder; what does the eighth pay?

17. How many tens and units will remain, if we subtract 7 tens from 11 tens and 8 units? 4 tens and 8 units are how many?

18. 9 tens + how many are 14 tens and 4 units?

19. 15 tens and 6 units — how many are 8 tens and 4 units?

20. 8 tens and 4 units + how many are 100?

21. 124 + 16 — 85 are how many?

22. 80 + 50 + 20 + 8 + 9 — 5 — 6 are how many?

23. 17 + 16 + 35 — 10 + 7 — 2 are how many ?

24. 40 + 30 + 70 + 6 + 7 — 45 are how many ?

25. 62 + 8 + 75 — 25 — 6 are how many ?

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DIVISION.

SECTION XII.

DIVISION is the process of finding into how many equal parts a given number can be divided, or of finding how many times one number is contained in another number of the same kind.

Division is the opposite of Multiplication, and as the latter is an extension of Addition, so, in like manner, Division may be regarded as an extension of Subtraction. To divide 9 by 3 is the same as finding the number of times that 3 may be subtracted from 9 ; first, we say, take 3 from 9 and 6 remains, then 3 from 6 and 3 remains, and lastly, 3 from 3 and nothing remains ; hence, we ascertain that 3 is contained in 9 three times without a remainder.

The number to be divided is called the *dividend*. The number divided by is called the *divisor*. The result of the operation is called the *quotient*. When anything remains after dividing, it is called the *remainder*.

The sign of Division is a horizontal line between two dots, ÷, and is read *divided by*.

1. How many units are contained in 5 ?

2. If a barrel of apples be worth 2 dollars, how many barrels of apples can you buy for 14 dollars ?

SOLUTION. As many times as 2 dollars are contained in 14 dollars, so many barrels of apples can I buy ;

2 dollars are contained in 14 dollars 7 times. Ans. 7 barrels.

3. How many times 2 in 14? in 16? in 22? in 28?

4. A man paid 24 dollars for calves, at 8 dollars a head; how many did he purchase?

5. How many times 8 in 24? in 40? in 48? in 64?

6. If 24 boys were separated into classes of 4 boys each, how many classes would be formed?

7. How many times 4 in 16? in 24? in 32? in 48?

8. If a cow be worth 5 sheep, how many cows can be bought for 25 sheep? how many for 40 sheep?

9. How many times 5 in 15? in 25? in 35? in 45?

10. If one yard of cloth be worth 5 dollars, how many yards can be bought for 50 dollars? how many for 35 dollars? for 45 dollars?

11. In an orchard there are 42 trees standing in rows, with 6 trees in a row; how many rows are there in the orchard?

12. How many times 6 in 42? in 54? in 18? in 60?

13. If one man can do a piece of work in 56 days, how many men will be required to do the same in 7 days?

14. How many times 7 in 49? in 21? in 35? in 63?

15. How many eggs can be bought for 63 cents, at the rate of a dozen for 9 cents?

16. Fifty-six are how many sevens? fifty-six are how many eights? sixty-three are how many sevens? sixty-three are how many nines?

17. How many times 9 in 18? in 36? in 54? in 72?

18. How long will it take a horse to travel 72 miles, if he travels 8 miles in each hour?

19. The sum of 45 dollars was divided, in prizes of 9 dollars each, among some sailors; among how many sailors was it divided?

20. If 90 nuts are distributed among some boys, each one receiving 9, how many boys are allowed to share?

21. At 10 cents per yard, how much calico can be purchased for 80 cents? how much for 100 cents?

22. 32 are how many times 2? 4? 8? 16?

23. 36 are how many times 2? 3? 4? 6? 9? 12? 18?

24. 48 are how many times 2? 3? 4? 6? 8? 12? 16?

25. 60 are how many times 2? 3? 4? 5? 6? 10? 12? 15?

26. 64 are how many times 2? 4? 8? 16?

27. 16 ÷ 8 are how many? 24 ÷ 6 are how many? 24 ÷ 4 are how many?

28. How many times 8 in 72? In 72 how many times 9? 80 ÷ 10 are how many? How many times 8 in 80?

29. How many times 12 in 84? how many times 7 in 84?

30. 12 divided by what number gives the quotient 6? 4? 3? 2?

31. 39 divided by what number gives the quotient 3? 13?

32. What numbers, multiplied together, will produce 12? 16? 15? 18? 24? 27? 36? 42? 48? 49?

33. How many times 3 tens in 12 tens? in 18 tens? in 24 tens? in 27 tens?

34. How many times 30 in 120? in 180? in 240? in 270?

35. How many times 6 tens in 36 tens? in 48 tens? in 54 tens? in 72 tens?

36. How many times 60 in 360? in 480? in 540? in 720?

37. What is Division?

38. What is the number to be divided called?

39. What is the number we divide by called?

40. What is the result of the operation called?

41. What is the number called that remains undivided, after the division is performed?

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SECTION XIII.

In the previous section questions were proposed in which the divisor was exactly contained in the dividend. In the present section the dividend will not exactly contain the divisor, and the remainder is given as a part of the answer: thus, 7 ÷ 3 gives 2 for the quotient, and 1 for the remainder.

1. How many times 8, and what remainder, in 11?

2. In a piece of cloth containing 15 yards, how many pieces, of 4 yards each, may be obtained, and how many yards will there be remaining?

3. How many times 4, and what remainder, in 17? in 23? in 27? in 33? in 37? in 45?

4. How many miles can you ride in a stage-coach for 27 cents, if you pay 5 cents for every mile, and how many cents will you have remaining?

5. How many times 5, and what remainder, in 18? in 21? in 29? in 33? in 42? in 47? in 53?

6. A gardener wishes to place 49 trees in rows of 6 trees each; how many rows will there be, and how many trees will he have remaining?

7. How many times 6, and what remainder, in 17? in 21? in 26? in 33? in 45? in 49? in 62? in 73?

8. If you have 53 shillings, to how many men can you give 7 shillings each, and what number of shillings will you have remaining?

9. How many times 7, and what remainder, in 13? in 19? in 25? in 31? in 38? in 43? in 50? in 58? in 69? in 76?

10. I have 47 pounds of tea in a chest; how many packages of 8 pounds can I put up, and how many pounds will remain in the chest?

11. How many times 8, and what remainder, in 17? in 23? in 36? in 42? in 50? in 59? in 65? in 76? in 90?

12. How many tons of coal, at 9 dollars a ton, can be bought for 85 dollars, and how many dollars will remain?

13. How many times 9, and what remainder, in 17? in 25? in 29? in 39? in 48? in 59? in 64? in 75? in 82? in 94?

14. John has 43 cents; to how many poor women can he give 10 cents each, and how many cents will he have left?

15. How many times 10, and what remainder, in 24? in 36? in 48? in 55? in 64? in 73? in 89? in 97?

16. A grocer has 92 gallons of oil to draw off into casks that will hold 11 gallons each; how many casks can he fill, and how many gallons will he have remaining?

17. How many times 11, and what remainder, in 25? in 37? in 45? in 59? in 64? in 78? in 89? in 96?

18. A gardener has 65 peaches which he wishes to send to market in baskets that will contain 12 peaches each; how many full baskets can he send, and how many peaches will he have remaining?

19. In a gross there are 12 dozen; how many gross in 105 dozen?

20. How many times 12, and what remainder, in 21? in 37? in 49? in 56? in 63? in 82? in 95? in 109? in 126?

21. How many yards of calico, at 13 cents a yard, can be purchased for 76 cents, and what number of cents will there be remaining?

MULTIPLICATION COMBINED WITH DIVISION.

SECTION XIV.

Obs. The sign of equality, =, is read *are*, or *are equal to*; as 2 and 2 = 4 is read, 2 and 2 are 4, and 8 × 5 ÷ 10 = 4 is read, 8 times 5 divided by 10 are equal to 4.

1. FROM the product of 6 times 4, how many times can 3 be subtracted?

2. How many times are 8 contained in 4 times 6?

3. Bought oranges at 4 cents each, and paid 4 ten-cent pieces for them; what number of oranges did I buy?

4. Sold 6 pounds of pilot-bread at 5 cents a pound; how many 3 cent pieces are required to pay for it?

5. 10 times 3 are how many times 5?

6. 6 times 6 are how many times 3?

7. A man purchased 10 chaldrons Sidney coal for 6 dollars a chaldron; how many 5 dollar gold pieces are required in payment?

8. How many times are 8 contained in 10 times 4? in 2 times 12? in 2 times 16?

9. A farmer exchanged 3 pounds of butter, worth 66 cents, for cocoa worth 6 cents a pound; how much cocoa did he receive?

10. How many times are 7 contained in 2 times 14? in 3 times 14? in 6 times 14?

11. Loaf-sugar, at 10 cents per pound, is given in exchange for 5 pounds of Java coffee, at 11 cents per pound, and 5 dozen of eggs, at 9 cents per dozen; how many pounds of sugar are given?

12. 5 times 8 are how many times 10?

13. Twelve inches make one foot; how many feet are there in 24 inches? in 3 times 16 inches? in 6 times 10 inches?

14. How many times 2 feet in 72 inches?

15. Bought 6 cords of wood at 7 dollars a cord; how many sheep, at 3 dollars a head, will be equal in value to the wood?

16. 6 times 7 are how many times 6? 3? 4? 5? 11? 14?

17. Four farthings make one penny; how many pence in 6 times 6 farthings?

18. 10 times 7 are how many times 11? 8? 10? 12? 6?

19. How many barrels of flour, at 6 dollars a barrel, are equal in value to 6 bags of coffee, at 12 dollars a bag?

20. 6 times 12 are how many times 6? 7? 8? 9? 10?

21. A tailor bought 11 yards of broadcloth, at 6 dollars a yard, and gave in payment vests worth 5 dollars apiece; how many vests did he give, and what number of dollars remained unpaid?

22. 11 times 6 are how many times 5? 8? 9? 11? 12?

23. A milkman bought 12 pounds of brown sugar, at 7 cents a pound, and paid for it with milk, at 6 cents a quart; how many quarts of milk did it require?

24. 12 times 7 are how many times 6? 8? 9? 10? 11? 12?

25. How many are $8 \times 12 \div 4$? SOLUTION. $8 \div 4 = 2$, therefore $8 \times 12 \div 4 = 2 \times 12 = 24$. Or $12 \div 4 = 3$, therefore $8 \times 12 \div 4 = 8$ times $3 = 24$. After becoming familiar with this process, the pupil will be able to give the answer at sight.

26. How many are $12 \times 7 \div 14$? SOLUTION. 14 is equal to 2 times 7; therefore, 12 multiplied by 7 divided by 14, is the same as 12 divided by 2, which is 6.

27. How many are $8 \times 14 \div 16$? $8 \times 14 \div 4$? $8 \times 14 \div 7$? $8 \times 14 \div 28$?

28. How many are $9 \times 12 \div 6$? $9 \times 12 \div 18$? $9 \times 12 \div 24$?

29. How many are $10 \times 12 \div 6$? $10 \times 12 \div 10$? $10 \times 12 \div 20$? $10 \times 12 \div 6$? $10 \times 12 \div 24$?

30. How many are $10 \times 16 \div 5$? $10 \times 16 \div 8$? $10 \times 16 \div 20$? $10 \times 16 \div 82$?

31. How many are $18 \times 24 \div 9$? $18 \times 24 \div 86$? $18 \times 24 \div 8$? $18 \times 24 \div 6$? $18 \times 24 \div 12$? $18 \times 24 \div 48$?

REVIEW.

SECTION XV.

1. BOUGHT coffee of four kinds; for Java I paid 12 cents a pound, for Mocha 13 cents, for Brazil 9 cents, and for St. Domingo 8 cents; what is the price paid for a package containing a pound of each kind?

2. How many are $100 - 64$? $100 - 49$? $100 - 86$? $100 - 16$? $100 - 9$? $100 - 57$?

3. How many are $5 + 4 + 8 + 7 + 6 + 9$?

4. How many are $5 + 7 + 5 + 8 + 9 + 6$?

5. How many are $4 + 8 + 6 + 9 + 11 + 12$?

6. 4 tens and 9 tens and 7 tens and 8 tens are how many tens? are how many?

7. If sperm candles are worth 50 cents, and tallow candles 13 cents a pound, what is the difference in their value?

8. $86 +$ how many are 100? $49 +$ how many are 100? $64 +$ how many are 100?

9. From 6 times 8 subtract 2 times 8. ANS. 4 times 8, or 32.

10. From 7 times 9 subtract 4 times 9.

11. From 11 times 11 subtract 4 times 11.

12. A boy received 90 credit-marks during the week; 19 on Monday, 21 on Tuesday, 4 on Wednesday, 16 on Thursday, and 23 on Friday; how many did he receive on Saturday?

13. How many are 23 + 17 + 25 + 19 — 70?

14. How many are 24 + 15 + 32 + 12 — 60?

15. How many are 35 + 16 + 23 + 9 — 50?

16. Bought 14 yards of cotton cloth at 10 cents a yard, and 2 yards of linen at 75 cents a yard; for which did I pay the most, and how much?

17. From 7 times 12 take 6 times 9?

18. From 8 times 11 take 5 times 6?

19. 3 times 8 + how many are 6 times 7?

20. If fish is worth 5 cents a pound, and beef 10 cents, how many pounds of fish will be equal in value to 7 pounds of beef?

21. How many times may 5 be subtracted from 75? Then 75 are how many times 5? 5 times 15 are how many?

22. Two men are 45 miles apart, and are travelling in the same direction; the foremost man at the rate of 6 miles an hour, and the other 9 miles an hour; in how many hours will the one overtake the other? how far will each man have travelled in the time?

23. Two men are 60 miles apart, and travel towards each other; one at the rate of 4 miles an hour, and the other 6 miles an hour; in how many hours will they meet? how far will each have travelled? what number of miles does one travel more than the other?

24. Two men are fifteen miles apart, and travel in opposite directions; one at the rate of 3 miles an hour, and the other 2 miles an hour; how far apart will they be in 8 hours? If they then turn and travel towards each other, in what time will they meet? How many hours, and what distance, will each man have travelled from the time he set out?

25. 5 times 3 and 6 times 3 and 8 times 3 are how many times 3? are how many?

26. 6 times 4 and 7 times 4 and 5 times 4 are how many times 4? are how many?

27. A man selects 90 sheep from his flocks in different pastures, taking 15 sheep from each pasture; from how many flocks does he select?

28. 4 tens and 3 tens are how many times 5? 6? 7? 8? 9? 11? 12? 13?

29. A shop-keeper bought a piece of cloth, containing 12 yards, for 6 dollars a yard; he sold 7 yards at 8 dollars a yard, but the remainder being damaged, he was obliged to sell it at 4'dollars a yard; did he gain or lose, and how much?

30. If from cloth worth 72 dollars, a tailor makes 3 suits of clothing, how much must he receive for each suit, in order to gain 12 dollars?

31. 5 times 5 and 6 times 5 and 7 times 5 less 11 times 5 are how many times 5? are how many?

32. 3 times 6 and 4 times 6 and 11 times 6 less 7 times 6 are how many times 6? are how many?

33. What numbers multiplied together will produce 25? 49? 56? 72? 21? 34? 66? 84? 132? 144?

34. What two numbers added together will produce 17? 21? 48? 35? 18? 43?

35. What three numbers added together will produce 17? 24? 48? 35? 18? 43? ∕

36. What four numbers added together will produce 17? 24? 48? 35? 18? 43?

37. 3 times 7 and 5 times 7 and 12 times 7 less 7 times 7 are how many times 7? are how many?

38. 4 times 8 and 6 times 8 and 13 times 8 less 9 times 8 are how many times 8? are how many?

39. What four numbers added together will produce 19? 25? 37? 41? 49? 55?

40. What five numbers added together will produce 19? 25? 37? 41? 49? 55?

PRIME AND COMPOSITE NUMBERS.

SECTION XVI.

A NUMBER which cannot be formed by multiplying together two or more whole numbers greater than a unit is called a *prime number;* as 1, 2, 3, 5, 7.

A number that can be formed by multiplying together two or more whole numbers greater than a unit is called a *composite number;* as 4, 6, 9, 8.

When a composite number is analyzed into prime numbers, the prime numbers are called its *prime factors.* Thus the prime numbers 2 and 3 are prime factors of 6, because $3 \times 2 = 6$. The prime factors of 8 are 2, 2, 2; because $2 \times 2 \times 2 = 8$.

The pupil will state which are prime and which composite numbers in the table, and analyze the composite numbers into their prime factors.

OBS. To find the prime factors of the composite numbers in the table, divide the *even* numbers by 2 ; the odd numbers by 3, 5, or 7. After the division has been performed once, the pupil will readily perceive what the other divisors must be, in order that all may be *prime* factors. Thus, the number 50 is an even number ; $50 \div 2 = 25$; the pupil will instantly perceive that the prime factors of 25 are 5, 5 ; therefore the prime factors of 50 are 2, 5, 5.

1	11	21	31	41
2	12	22	32	42
3	13	23	33	43
4	14	24	34	44
5	15	25	35	45
6	16	26	36	46
7	17	27	37	47
8	18	28	38	48
9	19	29	39	49
10	20	30	40	50

51	61	71	81	91
52	62	72	82	92
53	63	73	83	93
54	64	74	84	94
55	65	75	85	95
56	66	76	86	96
57	67	77	87	97
58	68	78	88	98
59	69	79	89	99
60	70	80	90	100

What is a prime number?
What is a composite number?
What are prime factors?

COMMON FACTORS.

SECTION XVII.

A NUMBER that will divide two or more numbers without a remainder is called a *common* factor of those numbers. The *greatest* number that will divide two or more numbers without a remainder is called the greatest common factor. Thus, 2, 3, and 6, are common factors of 12 and 18, but 6 is the *greatest* common factor of these numbers.

OBS. To find the greatest common factor of two numbers, divide the greater number by the smaller, and the smaller by the remainder, and thus continue dividing the last divisor by the last remainder, till nothing remains ; the divisor used last of all is the greatest common factor. Thus, to find the greatest common factor of 12 and 18, divide the greater number (18) by the smaller number (12), and the smaller number (12) by the remainder (6), and, as there is no remainder, 6, the last divisor, is the greatest common factor.

Obs. The greatest common factor is also called the greatest *common measure,* or *common divisor.*

1. What is the greatest common factor of 12 and 16?

2. What is the greatest common factor of 4 and 8?

3. What is the greatest common factor of 8 and 12?

4. What is the greatest common factor of 15 and 20?

5. What is the greatest common factor of 12 and 18?

6. What is the greatest common factor of 21 and 28?

7. What is the greatest common factor of 16 and 28?

8. What is the greatest common factor of 14 and 35?

9. What is the greatest common factor of 18 and 27?

10. What is the greatest common factor of 16 and 40?

11. What is the greatest common factor of 24 and 30?

12. What is the greatest common factor of 22 and 44?

13. What is the greatest common factor of 28 and 63?

14. What is the greatest common factor of 24 and 32?

15. What is the greatest common factor of 27 and 81?

16. What is the greatest common factor of 18 and 45?

17. What is the greatest common factor of 32 and 48?

18. What is the greatest common factor of 64 and 72?

19. What is the greatest common factor of 39 and 52?

20. What is the greatest common factor of 64 and 96?

21. What is the greatest common factor of 132 and 144?

22. What is the greatest common factor of 21 and 35?

23. What is the greatest common factor of 35 and 63?

24. What is the greatest common factor of 108 and 132?

25. What is the greatest common factor of 60 and 96?

OBS. When there are more than two numbers, first find the greatest common factor of any two of the numbers, and then of that factor and another of the numbers, and thus continue till all the numbers have been taken.

26. How do you find the greatest common factor of two numbers?

27. When there are more than two numbers, how is the greatest common factor found?

LEAST COMMON MULTIPLE.

SECTION XVIII.

THE least number that can be divided by two or more numbers without a remainder is called the *least common multiple* of those numbers. Thus, 12 is the least number that will exactly contain 3, 4, and 6; therefore 12 is the least common multiple of 3, 4, and 6.

To find the least common multiple of *two* numbers, divide one of the numbers by their greatest common fac-

tor, and multiply the quotient by the other number; the product will be the least common multiple.

1. What is the least common multiple of 6 and 8? The greatest common factor of 6 and 8 is 2; 6 ÷ 2 = 3; 3 × 8 = 24. ANS. 24 is the least common multiple.

2. What is the least common multiple of 12 and 16?

3. What is the least common multiple of 4 and 8?

4. What is the least common multiple of 8 and 12?

5. What is the least common multiple of 15 and 20?

6. What is the least common multiple of 12 and 18?

7. What is the least common multiple of 21 and 28?

8. What is the least common multiple of 16 and 28?

9. What is the least common multiple of 14 and 35?

10. What is the least common multiple of 18 and 27?

11. What is the least common multiple of 16 and 40?

12. What is the least common multiple of 24 and 30?

13. What is the least common multiple of 22 and 44?

14. What is the least common multiple of 28 and 63?

15. What is the least common multiple of 24 and 32?

16. What is the least common multiple of 27 and 81?

17. What is the least common multiple of 18 and 45?

18. What is the least common multiple of 32 and 48?

19. What is the least common multiple of 64 and 72?

20. What is the least common multiple of 39 and 52?

21. What is the least common multiple of 64 and 96?

22. What is the least common multiple of 21 and 35?

23. What is the least common multiple of 35 and 63?

When there are *more* than two numbers, first find the least common multiple of any two of the numbers, and then of this multiple and another of the numbers, and thus continue till all the numbers have been taken.

24. What is the least common multiple of 8, 10, and 12? The greatest common factor of 10 and 12 is 2; $10 \div 2 = 5$; $5 \times 12 = 60$; the greatest common factor of 8 and 60 is 4; $8 \div 4 = 2$; $2 \times 60 = 120$. ANS. 120.

When any of the numbers are factors of any other of the given numbers, they may be omitted in the operation.

25. What is the least common multiple of 10, 12, 24, and 30? 10 is a factor of 30, and 12 is a factor of 24, therefore we omit them in the operation; the greatest common factor of 24 and 30 is 6; $24 \div 6 = 4$; $4 \times 30 = 120$. ANS. 120.

26. What is the least common multiple of 6, 8, 12, and 16?

27. What is the least common multiple of 5, 7, 10, and 21?

28. What is the least common multiple of 6, 9, 18, and 24?

29. What is the least common multiple of 3, 4, 6, and 8?

30. What is the least common multiple of 14, 21, and 28?

31. What is the least common multiple of 5, 10, and 15?

32. What is the least common multiple of 8, 12, and 16?

33. What is the least common multiple of 7, 14, and 21?

34. What is the least common multiple of 4, 12, and 24?

35. What is the least common multiple of 8, 5, and 11?

36. What is the least common multiple of 3, 5, and 7?

37. What is the least common multiple of 18, 36, 72, and 108?

38. What is meant by the least common multiple?

39. How do you find the least common multiple of two numbers?

40. How do you find the least common multiple when there are more than two numbers?

41. When any of the numbers are factors of any other of the given numbers, what may be done?

FRACTIONS.

SECTION XIX.

A FRACTION is one or more equal parts of a unit.

Fractions are of two kinds, *common* and *decimal*. A common fraction is composed of two numbers, one written above the other, with a line between them. Thus, $\frac{4}{5}$ (four fifths), $\frac{3}{7}$ (three sevenths). The number below the line is called the *denominator*, and shows the number of equal parts into which the unit is divided. The number above the line is called the *numerator*, and shows how many parts are expressed by the fraction.

A fraction may also be considered as the quotient resulting from division, the numerator being the dividend, and the denominator the divisor. Thus, $\frac{4}{7}$ is the quotient resulting from 4 divided by 7, and may be read one seventh of four, or four sevenths of one.

·A fraction is increased when the numerator is made larger; for the larger the numerator, the more fractional parts there are taken. Thus, $\frac{4}{5}$ are greater than $\frac{3}{5}$. A fraction is increased when the denominator is made· smaller; for the smaller the denominator, the fewer are the number of parts into which the unit is divided. Thus, $\frac{1}{4}$ is greater than $\frac{1}{6}$.

· A fraction is made smaller when the numerator is made smaller, or when the denominator is made greater.

1. Into how many parts is the unit divided when the denominator of the fraction is 2?

2. What is meant by $\frac{1}{2}$ of an apple? ANS. One of the two equal parts into which an apple (unit) is divided.

3. How many halves make a whole? Why? Because the unit is divided into *two* equal parts to make halves, and the two equal parts taken together must make the unit, or whole.

4. Which is the greater, $\frac{1}{2}$ or $\frac{1}{3}$? Why? Because $\frac{1}{2}$ is one of the two equal parts, while $\frac{1}{3}$ is one of the three equal parts, into which a unit is divided.

5. How many thirds are there in one?

6. How many thirds are there in two? ANS. One contains $\frac{3}{3}$, therefore 2 will contain $\frac{3}{3} + \frac{3}{3} = \frac{6}{3}$.

7. How do you cut an apple so as to obtain $\frac{2}{3}$? ANS. I cut an apple into 3 equal pieces, then 2 of the pieces are $\frac{2}{3}$ of the apple.

8. How many fourths in 1? in 2? in 4?

9. What do you mean by $\frac{1}{4}$? ANS. One of the four equal parts into which a unit is divided.

10. What do you mean by $\frac{3}{4}$? ANS. Three of the four equal parts into which a unit is divided.

11. Into how many parts is the unit divided when the denominator is 4?

12. $\frac{3}{4}$ are how many times the fourth of one? ANS. Three times the fourth one.

13. If a class consists of 10 pupils, how many pupils are there in one half of the class? in one fifth of the class? in four fifths of the class?

14. What is one half of 10? 14? 18? 24?

15. What are five halves of 10? 14? 18? 24?

16. If 5 is one half of some number, what are 2 halves of the same number? 7 halves? 9 halves? 12 halves?

17. If 5 is one half of some number, how many halves of the same number are 15? 25? 45? 55?

18. If 15 are three halves of some number, how many halves of the same number are 20? 30? 40? 50?

19. Which is greater, $\frac{2}{3}$ or $\frac{3}{4}$? Why?

20. How many fifths in one? in two? in seven?

21. What do you mean by $\frac{4}{5}$ of an orange?

22. $\frac{4}{5}$ are how many times the fifth of one?

23. How do you cut a yard of cloth so as to obtain $\frac{4}{5}$ of it?

24. If an arithmetic costs 15 cents, what is $\frac{1}{3}$ of the cost of the book? what are $\frac{2}{3}$ of the cost of the book?

25. What is one third of 18? 27? 33? 42?

26. What are four thirds of 18? 27? 33? 42?

27. If 6 nuts are one third of the number of nuts that Charley has, what are two thirds of his nuts? How many nuts has he?

28. If 6 is one third of some number, how many thirds of the same number are 24? 48? 60? 72?

29. If 24 are four thirds of some number, how many thirds of the same number are 30? 42? 54? 66?

30. Which is the greater, $\frac{2}{3}$ or $\frac{3}{4}$? Why?

31. How many sixths in one? in two? in eight?

32. What do you mean by $\frac{3}{6}$? $\frac{3}{6}$ are how many times the sixth of one?

33. How would you obtain $\frac{4}{6}$ of a pine-apple?

34. If a sleigh cost 32 dollars, what is $\frac{1}{4}$ of the cost of the sleigh? what are $\frac{2}{4}$ of the cost of the sleigh? $\frac{3}{4}$ of the cost?

35. What are five fourths of 24? 36? 48? 52?

36. James has 9 cents, which is one fourth as much money as Edwin has; what is 3 fourths of Edwin's money? how much money has he?

37. If 3 is one fourth of some number, how many fourths of the same number are 18? 24? 27? 8? 6?

38. If 18 are 6 fourths of some number, how many fourths of the same number are 15? 38? 39? 45?

39. How many sevenths in one? in five? in nine?

40. What do you mean by $\frac{4}{7}$? $\frac{4}{7}$ are how many times the seventh of one?

41. Five gentlemen agreed to furnish a family of poor children with clothes and books, in order that they might attend school; the expense was 35 dollars, which they shared equally; what part of the money, and how much, did each pay?

42. What are seven fifths of 15? 20? 25? 45?

43. If 6 is one fifth of 30, what are four fifths of the same number? 6 fifths? 8 fifths? 12 fifths? 16 fifths?

44. Twelve melons are 4 fifths of the number of melons in my garden; how many times one fifth of my melons are 18 melons? 21 melons? 27 melons? 36 melons?

45. A market-man carried 42 ears of green corn to market, which were 6 fifths of the number of ears carried the day before; required 3 fifths of the number of ears carried the day before; 8 fifths of the number; 11 fifths; 13 fifths.

46. How many eighths in one?

47. What do you mean by $\frac{5}{8}$? $\frac{5}{8}$ are how many times the eighth of one?

48. I have 32 oranges to divide equally among 8 boys; what number of oranges can I give to each boy?

49. A gentleman has 66 acres of land in 6 fields of equal size; how many acres in 5 of the fields?

50. What are 8 sixths of 48? 60? 72? 84?

51. An angler being asked how many fish he had caught that morning, replied, that 18 were 8 sixths of the number taken; how many had he caught?

52. If 32 are 8 sixths of some number, how many sixths of the same number are 28? 36? 44? 52?

53. Lucy has 12 cherries, which are 4 sixths of the number she has eaten; what are 3 sixths of the number of cherries she has eaten? how many has she eaten?

54. How many ninths in one? in seven? in eleven?

55. What do you mean by $\frac{1}{5}$?

56. If 20 dollars are paid for 10 sevenths of a ton of hay, what is the price per ton?

57. What are 9 sevenths of 21? 35? 49? 56? 63?

58. Eighteen scholars are 2 sevenths of the number in the school; how many sevenths of the number are 36? 72? 81? 108?

59. A company of children went out to gather pond-lilies, and, upon returning, found that 63 were 9 sevenths of the number gathered; how many did the little girl receive that had 2 sevenths of the lilies for her share? how many did they gather?

60. How many tenths in one? in eight? in twelve?

61. If an acre of land will produce 24 bushels of corn, how much will 7 eighths of an acre produce?

62. What are 9 eighths of 48? 64? 96? 112?

63. Sixty-three credit marks are 9 eighths of the highest number of marks given to any boy; how many eighths of the number of marks does the boy receive who gets 49? how many eighths has the scholar that has 56?

64. If 49 are 7 ninths of some number, what are 9 ninths of the same number? 11 ninths? 14 ninths?

65. If 21 are 3 elevenths of some number, how many elevenths of the same number are 49? 63? 77? 91?

66. What is a fraction?

67. How are the numbers of a common fraction written?

68. What is the lower number called? What does it show?

69. What is the upper number called? What does it show?

70. How is a fraction made greater? How is a fraction made smaller?

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SECTION XX.

1. What part of 2 is 1? Ans. $\frac{1}{2}$. Why? Because 1 must be the half of 2 times 1.

2. What part of 3 is 1? is 2?

3. What part of 4 is 1? is 2? is 3?

4. What part of 5 is 1? is 2? is 3? is 4?

5. How many are 8 times the half of 2?

6. If 2 pounds of sugar cost 14 cents, what will 1 pound cost? 6 pounds? 8 pounds? 12 pounds?

7. If 3 pairs of shoes cost 6 dollars, what is the cost of 1 pair? 8 pairs? Solution. If 3 pairs of shoes cost 6 dollars, 1 pair will cost a third of 6 dollars, or 2 dollars; 8 pairs will cost 8 times 2 dollars, or 16 dollars. Ans. 16 dollars.

8. What are 10 times the half of 14? what are 9 times the third of 6?

9. If 4 cords of wood cost 24 dollars, what will 12 cords cost?

10. What are 9 times the fourth of 24?

11. How many bushels of oats will 9 horses eat in a month, if 6 horses eat 30 bushels in the same time?

12. How many are 8 times the sixth of 30?

13. If 8 cows are worth 160 dollars, how much are 7 of them worth?

14. What part of 8 is 1? is 7? is 5?

15. How many panes of glass are there in a window of your school-room? what part of the whole will 4 panes be?

16. How many scholars are reciting in your class? what part of the class are 4 scholars? 6 scholars? 10 scholars?

17. What will cost 10 pounds of butter, if 6 pounds
from the same firkin cost 108 cents? what will 5
pounds cost?

18. If 7 men earn 63 dollars in a week, what will
9 men earn in the same time?

19. What number is 10 times the twelfth of 108?

20. What number is 9 times the seventh of 63?

21. If two gallons of molasses cost 60 cents, what
will seven gallons cost, at the same rate?

22. What cost 19 pounds of rice, at 65 cents for 13
pounds?

23. What will 28 oranges cost, at 36 cents a dozen?

24. If seven pounds of cask-raisins cost 77 cents,
what will be the cost of twelve pounds of the same?

25. What part of 7 is 12?

26. How many are 7 times the half of 60? 21 times
the thirteenth of 65? 28 times the twelfth of 36?

27. A man wishes to divide 6 times the half of 24
cents equally among 9 boys; how many cents must he
give to each boy?

28. Henry, James and Joseph, have 4 times the
third of 27 filberts to share equally; what was Joseph's
share?

29. Five times the fourth of 48 sheep are owned in
equal shares by 10 persons; how many sheep has each
person?

30. A tailor spends 40 dollars for vestings, and 7
times the fifth of that sum for broadcloth, at 4 dollars a
yard? how many yards of broadcloth does he buy?

31. In a basket there were 9 times the sixth of 36
peaches, and the owner put in 12 more; they were all
purchased by a teacher, and distributed in equal shares
among 11 studious boys; how many peaches did one
boy receive? four boys? ten boys?

32. A gardener carries to market strawberries and
cherries: he sells the former at 28 cents a box, nine
times the seventh of which is 18 cents less than what

he receives for 9 quarts of cherries; what does he get for a quart of cherries?

33. A gentleman received 24 dollars, in premiums from the Horticultural Society, for specimens of fruits and flowers, ten times the eighth of which sum was 6 dollars more than he paid for 8 young pear trees; what did he pay apiece for the pear trees?

34. What number, multiplied by 5, will produce 6 times the half of 30?

35. What number, multiplied by 12, will produce 9 times the fourth of 32?

36. What number, multiplied by 8, will produce 10 times the sixth of 72?

37. What number, multiplied by 11, will produce 6 times the eighth of 72, less 10?

38. How many times 6 are there in 7 times the tenth of 80, plus 10?

39. What number, divided by 4, will give 5 more than 5 times the twelfth of 108?

40. What number, divided by 3, will give 12 less than 3 times the fourth of 56?

41. What number, divided by 5, will give 3 times the ninth of 90?

42. What number, divided by 6, will give 4 times the third of 24, plus 8?

43. What number, multiplied by 9, will produce 8 times the fifth of 40, plus 17?

44. How many times 7 are there in 12 times the eleventh of 44, plus 15?

45. There are four pastures, each containing 2 less than three times the ninth of 45 sheep; how many sheep are there in all?

46. How many are 6 times the eighth of 40, and 5 times the sixth of 30? What remains, if 9 times the seventh of 35 be subtracted from the sum?

47. A farmer has 80 sheep; into how many pastures

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must he put them, in order to have in each pasture **5** more than 3 times the fifth of 25 sheep?

48. What number is that, which, being multiplied by the half of 18, will produce 19 times the fifth of **10**, plus 7?

49. What number, divided by 6, will give 15 times the twelfth of 48?

50. What number, added to 4 times the eighth of 56, will make the sum 30?

SECTION XXI.

1. How many times are 2 contained in 5? Ans. Two and a half times, because $2 \times 2 = 4$, and 1 is the half of 2.

2. How many are 3 and a half times 2? Ans. 7, because 3×2 are 6, and the half of 2 is 1, and $6 + 1 = 7$.

3. How many times 3 in 10? in 8?

4. How many are 4 times 3, and twice the third of 3?

5. How many times 4 in 21? in 23? in 27?

6. How many must I add to 9 times 4 to make 38?

7. If 3 times the fourth of 4 be taken from 6 times 4, what number will remain?

8. What number will remain, if I subtract 2 times 4 from 5 times 4, and 3 times the fourth of 4?

9. What is the sum of 2 times 5, and 3 times the fifth of 5?

10. William spent 5 cents, and his brother spent three and 2 fifths times as much; how much did his brother spend?

11. How many times 5 in 11? in 17? in 23?

12. How many are 4 times 5, and 3 times the fifth of 5?

13. How many are 7 times 5, and 4 fifths of 5?

14. What number will remain, if I subtract 3 times the fifth of 5 from 9 times 5, and 2 fifths of 5?

15. If a boy can earn 7 dollars in one week, how much can he earn in 5 weeks and 2 sevenths of a week?

16. How many times 7 in 11? in 23?

17. How many are 8 times 7, and 3 sevenths of 7?

18. What number will remain, if I subtract 4 times the seventh of seven from 6 times 7, and 5 sevenths of 7?

19. How many must I add to 9 times 7, and 6 sevenths of 7, to make 72?

20. A man paid 8 dollars for a firkin of butter; what would be the cost of 8 firkins and 3 eighths of a firkin of butter, at the same rate?

21. How many times 8 in 39? in 63?

22. What is the sum of 8 times 8, and 7 times the eighth of 8?

23. What number will remain, if I subtract 7 times the eighth of 8 from 9 times 8, and 5 eighths of 8?

24. How many must I add to 3 times 8, and 4 eighths of 8, to make 42?

25. If a keg of molasses costs 9 dollars, what will 7 kegs and 5 ninths of a keg of molasses cost? what will 11 kegs and 7 ninths of a keg of molasses cost?

26. How many are 12 times 9, and 3 ninths of 9?

27. In 5 times 9, and 8 ninths of 9, how many times 7? 8? 11? 13?

28. Twice 9 and 4 times 9, and 6 ninths of 9, are how many times 9, and what part of 9? how many times 6? 5? 3? 2?

29. How many must I add to 3 times 9 and 5 times 9, and 7 ninths of 9, to make 100? 120? 150?

30. What number will remain, if I subtract 2 times 9, and 3 ninths of 9, from 5 times 9, and 5 ninths of 9? from 5 times 8, and 5 eighths of 8?

31. A church-spire is 12 and 5 tenths times the

length of a pole 10 feet long; what is the height of the spire ?

32. How many are 8 times 10, and 5 tenths of 10 ?

33. How many times 10 in 72 ? 120 ? 150 ? 180 ?

34. How much must Edwin earn, to have 100 cents, if he has already earned 7 times 10 cents, and 9 tenths of 10 cents ?

35. Edwin earned 7 times 10 cents, and 9 tenths of 10 cents; and he spent 2 times 10 cents, and 5 tenths of 10 cents for a new reading book; how much must he obtain, in order to have 100 cents ?

36. Six times 10 and 5 times 10, and 7 tenths of 10, are how many times 10 and what part of 10 ? how many times 12 ? 11 ? 9 ?

37. What will remain, if I subtract 3 times 10, and 9 tenths of 10, from 5 times 9, and 4 ninths of 9 ? from 8 times 8, and 7 eighths of 8 ?

38. A laborer saves 11 dollars from his wages each month; how much will he save in 6 months and 8 elevenths of a month ? If, at the expiration of that time, he spends 2 times 11 dollars, and 1 eleventh of 11 dollars, for a suit of clothes, how much will he have remaining ?

39. Seven times 11 and 6 times 11, and 7 elevenths of 11, are how many times 11, and what part of 11 ? 10 ? 15 ?

40. If a village school consists of 6 times 12 and 7 twelfths of 12 pupils, and 2 times 12 and five twelfths of 12 pupils, are detained at home by a severe snowstorm, how many times 12 and what part of 12 pupils were present at school ? how many were present ?

41. Six times 12, and 7 twelfths of 12, less 15, are how many times 2 ? 4 ? 8 ? 16 ?

42. When coffee is worth 13 cents a pound, what will cost 8 pounds and 4 thirteenths of a pound ? how many times 12 cents will pay for it ?

43. When figs are worth 12 cents a pound, what will cost 8 pounds and 4 twelfths of a pound?

44. How many are 7 times 12, and 8 twelfths of 12? 9 times 12, and 9 twelfths of 12?

45. How many are 6 times 13, and 7 thirteenths of 13? 7 times 13, and 5 thirteenths of 13?

46. How many are 5 times 12, and 5 twelfths of 12, plus 4 times 13, and 3 thirteenths of 13?

47. 2 times 12, and 3 times 12, plus 2 times 13, and 4 times 13, are how many?

48. A boy has 7 times 14 cents, plus 2 fourteenths of 14 cents, and he wishes to buy a pair of skates, which are worth 1 dollar and 56 cents; how many cents must he obtain, in order to make the purchase?

49. How many are 4 times 12, and 4 times 13, and 5 times 14, minus 4 times 15?

NUMERATION.

SECTION XXII.

NUMERATION is the art of expressing any number whatever by figures. There are ten figures: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0, and they are used to express quantity. The first nine are called significant figures, because they have a value assigned them; the cipher has no representative value, and is only used to fill a place not occupied by any significant figure.

Figures are written in different places; and have different values assigned them, according to the places which they occupy. The first place, which is always at the right, is the units' place; the second is the tens' place; the third is the hundreds' place, &c.

The figure 1, in the units' place, expresses 1 unit; in the tens' place, 1 ten, or 10 units; in the hundreds' place, 1 hundred, or 10 tens, or 100 units, &c. Any figure, moved one place to the left, expresses 10 times its former value; two places to the left, 100 times its former value.

The figures expressing a number may be divided into periods of three figures each, as in the following table:

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
6	4	5	7	3	2	9	6	4

This is enumerated thus: 4 units, 6 tens, 9 hundreds, 2 thousands, 3 tens of thousands, 7 hundreds of thousands, 5 millions, 4 tens of millions, 6 hundreds of millions. It is read thus: six hundred and forty-five millions, seven hundred and thirty-two thousand, nine hundred and sixty-four.

3d Period.   2nd Period.   1st Period.

1. What does the figure 3 express in the first place? in the second place? in the fourth place?

2. What does the figure 6 express in the third place? in the sixth place? in the ninth place?

3. In what place must 5 be written, to express fifty? to express five thousand? to express fifty millions?

4. How must we write the 4 and the 5 to express four hundred and five, and what must we put in the tens' place?

Obs. The pupil must be exercised upon questions like the above, until perfectly familiar with them.

5. Enumerate and read the following numbers:

24	584	57856
568	6392	843279
4397	29528	25344853

6. How would you express in figures the number five thousand seven hundred and four? Ans. I would place 5 in the thousands' place, 7 in the hundreds' place, 0 in the tens' place, and 4 in the units' place.

7. Tell in what manner you would express in figures the following numbers: Twenty-five. Thirty-nine. Ninety-nine. Three hundred and thirty. Three hundred and three. Three thousand and thirty. Three thousand and three.

8. Tell in what manner you would express in figures the following numbers: Forty-five thousand and fifty-five. Five hundred thousand and five. Four thousand four hundred and four. Eighty-four millions four thousand and eighty-four.

Obs. The teacher will multiply questions, at pleasure.

9. What is numeration?

10. What are figures? what is their use?

11. Which are significant figures? why?

12. Has the cipher any value? how is it used?

13. Where do we begin to enumerate?

14. What does 1 express in the units' place? what in the tens' place? in the hundreds' place? in the thousands' place?

15. How does it affect the value of a figure to move it to the left?

16. Repeat the numeration table as far as hundreds of millions.

17. What is the name of the second place? third place? fifth place?

18. Which is the thousands' place? hundreds of thousands' place?

SECTION XXIII.

REDUCTION OF FRACTIONS TO THEIR LOWEST TERMS.

WE do not change the value of a fraction when we multiply or divide both the numerator and denominator by the same number; for, if we multiply the denominator of a fraction by 2, 3, or 4, we make the fraction $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$ of its first value; so, on the other hand, if we multiply the numerator by 2, 3, or 4, we increase the fraction as much as we decreased it in the former case. Again, if we divide the denominator of a fraction by 2, 3, or 4, we make the fraction 2, 3, or 4 times its first value; so, on the other hand, if we divide the numerator by 2, 3, or 4, we make the fraction as much smaller as we increased it in the former case. See Section XIX.

We reduce a fraction to its *lowest terms* by dividing both numerator and denominator by their greatest common factor.

A fraction is expressed in its lowest terms when no number greater than a unit will divide *both* of its terms without a remainder.

1. Reduce $\frac{2}{4}$ to its lowest terms.
2. Reduce $\frac{6}{8}$ to its lowest terms.
3. Reduce $\frac{8}{10}$ to its lowest terms.
4. Reduce $\frac{10}{12}$ to its lowest terms.
5. Reduce $\frac{12}{24}$ to its lowest terms.
6. Reduce $\frac{18}{36}$ to its lowest terms.
7. Reduce $\frac{24}{48}$ to its lowest terms.
8. Reduce $\frac{35}{45}$ to its lowest terms.
9. Reduce $\frac{66}{77}$ to its lowest terms.
10. Reduce $\frac{11}{55}$ to its lowest terms.
11. Reduce $\frac{26}{39}$ to its lowest terms.
12. Reduce $\frac{88}{63}$ to its lowest terms.
13. Reduce $\frac{64}{56}$ to its lowest terms.

14. Reduce $\frac{96}{112}$ to its lowest terms.
15. Reduce $\frac{18}{32}$ to its lowest terms.
16. Reduce $\frac{18}{27}$ to its lowest terms.
17. Reduce $\frac{64}{72}$ to its lowest terms.
18. Reduce $\frac{24}{48}$ to its lowest terms.
19. Reduce $\frac{12}{45}$ to its lowest terms.
20. Reduce $\frac{27}{81}$ to its lowest terms.
21. Reduce $\frac{24}{32}$ to its lowest terms.
22. Reduce $\frac{28}{63}$ to its lowest terms.
23. Reduce $\frac{22}{44}$ to its lowest terms.
24. Reduce $\frac{84}{44}$ to its lowest terms.
25. Reduce $\frac{48}{48}$ to its lowest terms.
26. Reduce $\frac{81}{108}$ to its lowest terms.
27. Reduce $\frac{75}{120}$ to its lowest terms.
28. Reduce $\frac{100}{150}$ to its lowest terms.
29. Reduce $\frac{112}{116}$ to its lowest terms.
30. How is a fraction reduced to its lowest terms?
31. Why is not the value of a fraction changed by reducing it to its lowest terms?
32. When is a fraction expressed in its lowest terms?

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SECTION XXIV.

REDUCTION OF IMPROPER FRACTIONS TO WHOLE OR MIXED NUMBERS, AND THE REDUCTION OF WHOLE OR MIXED NUMBERS TO IMPROPER FRACTIONS.

A *proper* fraction is one whose numerator is *less* than its denominator, as $\frac{2}{3}$ (2 thirds).

An *improper* fraction is one whose numerator is *equal to* or *greater* than its denominator, as $\frac{4}{4}$ (4 fourths), $\frac{9}{7}$ (9 sevenths).

OBS.   Proper and improper fractions are also called *simple fractions.*

A *mixed number* consists of a whole number and a fraction, as $5\frac{1}{2}$ (5 and one half).

The whole numbers are extracted from an improper fraction by dividing the numerator by the denominator for the units, and then annexing the remainder in the form of a fraction for the complete value. Thus, $\frac{17}{5} = 3\frac{2}{5}$, and conversely the *mixed number*, $3\frac{2}{5}$, is reduced to a fraction by multiplying the whole number (3) by the denominator of the fraction (5), and then adding the product to the numerator of the fraction (2), for the number of fifths in the proposed number.

1. What kind of a fraction is $\frac{5}{2}$? ANS. An improper fraction. Why?

2. How do you find the number of units in $\frac{5}{2}$? ANS. I divide 5 by 2, because there are $\frac{2}{2}$ in a whole one, and, therefore, there will be as many units as there are $\frac{2}{2}$ in $\frac{5}{2}$.

3. What kind of a fraction is $\frac{8}{3}$?

4. How many units in $\frac{8}{3}$?

5. Reduce $\frac{21}{3}$ to a whole number.

6. Reduce $\frac{27}{4}$ to a whole or mixed number.

7. In $\frac{26}{5}$ how many units?

8. How do you find the number of units in $\frac{32}{5}$?

9. How many units are contained in $\frac{37}{5}$?

10. Reduce $\frac{61}{4}$ to a whole or mixed number.

11. Reduce $\frac{42}{4}$ to a whole number.

12. How many halves in $3\frac{1}{2}$? ANS. $\frac{7}{2}$. Because in 1 there are $\frac{2}{2}$, in 3 there are 3 times $\frac{2}{2} = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = \frac{6}{2}$; $\frac{6}{2} + \frac{1}{2} = \frac{7}{2}$.

13. How many thirds in $4\frac{1}{3}$?

14. How many fourths in $5\frac{1}{4}$?

15. How many fourths of an apple in $7\frac{3}{4}$ apples?

16. To what number of children could I give $\frac{1}{4}$ of an orange each, if I had $8\frac{3}{4}$ oranges?

17. How many fifths in $11\frac{1}{5}$?

18. How many sixths in $12\frac{1}{6}$?

19. How many sevenths in $7\frac{4}{7}$?

20. How many sevenths in $18\frac{1}{7}$?

21. Reduce $9\frac{1}{7}$ to an improper fraction.

22. A merchant sold $10\frac{2}{3}$ yards of cloth; how many thirds of a yard did he sell?

23. A grocer sold $\frac{54}{8}$ of a pound of coffee; what number of pounds did he sell?

24. In $\frac{64}{8}$ how many units?

25. In $\frac{48}{9}$ how many units?

26. In $\frac{76}{8}$ how many units?

27. In $\frac{91}{9}$ how many units?

28. In $\frac{87}{7}$ how many units?

29. In $\frac{88}{12}$ how many units?

30. In $\frac{84}{8}$ how many units?

31. Reduce $9\frac{5}{6}$ to an improper fraction.

32. Reduce $5\frac{3}{8}$ to an improper fraction.

33. Reduce $7\frac{7}{8}$ to an improper fraction.

34. How many yards of ribbon can be purchased for $9\frac{3}{10}$ dollars, if each yard costs $\frac{1}{10}$ of a dollar?

35. In $10\frac{1}{11}$ how many elevenths?

36. In $16\frac{1}{2}$ how many halves?

37. In $30\frac{1}{4}$ how many fourths?

38. Elizabeth purchased $\frac{196}{9}$ of a yard of silk for a dress; what number of yards did she buy?

39. Reduce 6, 7, 8, 9, 10, and 11, to improper fractions, the teacher naming the denominator of the fraction to which he wishes the number reduced.

40. A boy found a purse containing 25 dollars; how many fifths of a dollar were there in the purse? how many thirds of a dollar? how many eighths? how many sixths?

41. The circumference of the wheel of a locomotive engine is $\frac{88}{12}$ of a foot; how many feet of the track does the engine pass over during each revolution of the wheel?

42. If an acre of land produces $27\frac{2}{3}$ bushels of corn, how many thirds of a bushel does it produce?

43. A farmer gathered $\frac{7.2}{1.8}$ of a bushel of apples from a tree in his orchard; how many bushels did he gather from the tree?

44. In 16$\frac{3}{4}$ yards of broadcloth, how many fourths of a yard?

45. A gentleman bought $\frac{9.2}{1.8}$ of a pound of honey; what number of pounds of honey did he purchase?

46. What is a proper fraction?

47. What is an improper fraction?

48. What is a mixed number?

49. How are the whole numbers extracted from improper fractions?

50. How are mixed numbers reduced to improper fractions?

SECTION XXV.

ADDITION AND SUBTRACTION OF FRACTIONS HAVING A COMMON DENOMINATOR.

FRACTIONS that are composed of the same parts of a unit are said to have a *common denominator*.

We add or subtract fractions of the same denomination by adding or subtracting the numerators, and then placing the common denominator under the sum or difference: for $\frac{2}{6}$ and $\frac{4}{6}$ make $\frac{6}{6}$, in the same manner that 2 quantities and 4 quantities of the *same kind* make 6 quantities of that kind. In like manner, 2 quantities of one kind, taken from 4 quantities of the same kind, leave 2 quantities of that kind.

When fractions have *different denominators* we cannot add them in that state, because the parts are of different magnitudes.

1. What is the sum of $\frac{1}{2}$ and $\frac{3}{2}$? SOLUTION. $\frac{1}{2} + \frac{3}{2}$ $= \frac{4}{2}$, or 2. ANS. 2.

2. What is the sum of $\frac{5}{2}$ and $\frac{7}{4}$? Ans. $\frac{7}{2}$ or $3\frac{1}{2}$.

3. What is the difference between $\frac{5}{2}$ and $\frac{3}{2}$? Ans. $\frac{2}{2}$ or 1.

4. What is the sum of $\frac{5}{6}$ and $\frac{6}{5}$?

5. What is the sum of $\frac{4}{4}$ and $\frac{2}{4}$?

6. If I divide an apple into three equal parts, and then give one of the parts away, what part of the apple have I remaining?

7. A post is $\frac{2}{6}$ of its length in the ground; what fractional part of the post is above ground?

8. Walter had $\frac{2}{4}$ of a pound of candy, and his sister had $\frac{1}{6}$ of a pound; how much must be given to them, in order that they may both together have 1 pound?

9. Two boys, having each an equal sum of money, agreed to keep but one purse, and to share equally the things that were bought; they spent $\frac{1}{10}$ of the money for confectionary, $\frac{2}{10}$ of it for oranges, and $\frac{3}{10}$ for marbles, and then, being tired of the arrangement, divided the remainder justly between them. What part of the money remained to be divided?

10. $\frac{3}{20}$ of a buoy were in the mud, and $\frac{9}{20}$ were in the water, before some mischievous sailors cut off the part that was above the water; what part of the buoy was cut off?

11. What is the sum of $2\frac{2}{3}$ and $3\frac{2}{3}$? Solution. $2\frac{2}{3}$ are equal to $\frac{8}{3}$; $3\frac{2}{3}$ are equal to $\frac{11}{3}$; $\frac{11}{3} + \frac{8}{3} = \frac{19}{3}$, or $6\frac{1}{3}$. Ans. $6\frac{1}{3}$.

12. What is the sum of $5\frac{3}{4}$ and $8\frac{2}{4}$? Solution. $\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$, or $1\frac{2}{4}$; $5 + 8 + 1\frac{2}{4} = 14\frac{2}{4}$. Ans. $14\frac{2}{4}$.

Obs. When the numbers are large, it will be easier for the pupil to add the fractions separately from the whole numbers.

13. What is the sum of $4\frac{2}{3}$ and $5\frac{5}{6}$?

14. What is the sum of $2\frac{2}{4}$ and $\frac{2}{3}$?

15. What is the sum of $9\frac{5}{6}$ and $8\frac{6}{8}$?

16. What is the sum of $10\frac{3}{8}$ and $\frac{5}{8}$?

17. A boy paid $12\frac{6}{12}$ cents for oranges, and $18\frac{9}{12}$ cents for lemons; what did he pay for both?

18. A farmer sold a load of wood for $7\frac{3}{5}$ dollars, and a ton of hay for $16\frac{2}{3}$ dollars; how much did he receive for both?

19. A market-man sold melons for $37\frac{1}{2}$ cents, and cucumbers for 25 cents; what did he receive for both?

20. What is the difference between $2\frac{1}{4}$ and $\frac{4}{7}$? Solution. $2\frac{1}{4}$ are equal to $\frac{7}{7}$; $\frac{7}{7} - \frac{4}{7} = \frac{3}{7}$ or 1. Ans. 1.

21. What number must be added to $1\frac{5}{8}$ in order that the sum may be $3\frac{5}{8}$?

22. What number must be added to $1\frac{5}{8}$ in order that the sum may be $4\frac{1}{8}$?

23. A trader, having $6\frac{1}{4}$ barrels of flour, sold $3\frac{5}{7}$ barrels of it; how much had he remaining?

24. A lady paid $5\frac{9}{16}$ dollars for a dress, and $3\frac{12}{16}$ dollars for a bonnet; what is the difference in their value? Solution. As I cannot subtract $\frac{12}{16}$ from $\frac{9}{16}$, I borrow 1 from the whole number, and subtract $\frac{12}{16}$ from that: $\frac{16}{16} - \frac{12}{16} = \frac{6}{16}$; $\frac{6}{16} + \frac{9}{16}$ (the fraction in the minuend) $= \frac{15}{16}$. As one was borrowed from the 5, but 4 remain: $4 - 3 = 1$. Ans. $1\frac{15}{16}$ dollars.

Obs. When the fraction in the subtrahend cannot be subtracted from the fraction in the minuend, subtract the numerator in the subtrahend from the denominator in the minuend. and to the remainder add the numerator in the minuend; the sum will be the numerator of the fraction in the remainder. The whole number in the minuend must be decreased by 1 before the subtraction is performed upon the whole numbers.

25. Emily purchased $4\frac{8}{16}$ yards of ribbon to trim her bonnet with, and $2\frac{9}{16}$ yards for a sash; how much ribbon did she purchase? How much more for her bonnet than for her sash?

26. Eliza paid $\frac{17}{20}$ of a dollar for lace edging, and $1\frac{7}{20}$ dollars for muslin; what did she pay for both? What was the difference in their value?

27. Ellen bought $4\frac{3}{12}$ yards of merino for a dress,

and $2\frac{7}{12}$ yards of colored cambric for the lining; how many yards were there in both? How many more in the dress than in the lining?

28. Abbie gave $4\frac{18}{25}$ dollars for a dress pattern of *mousseline-de-laine*, and $2\frac{21}{25}$ dollars for the trimmings for it; she then paid $1\frac{17}{25}$ dollars for having it made; what was the whole cost of the dress? How much more did she pay for the *mousseline* than for the trimmings and making?

29. Annie's cloak cost $16\frac{7}{8}$ dollars, and her bonnet cost $5\frac{3}{8}$ dollars; what did her cloak and bonnet together cost? What is the difference in their value?

30. What is the sum of $\frac{5}{7} + \frac{10}{7} + \frac{9}{7}$?

31. What is the sum of $\frac{6}{8} + \frac{7}{8} + \frac{8}{8}$?

32. What is the sum of $\frac{7}{9} + \frac{8}{9} + \frac{9}{9}$?

33. What is the sum of $\frac{5}{12} + \frac{8}{12} + \frac{10}{12} + \frac{9}{12}$?

34. What is the sum of $\frac{7}{16} + \frac{8}{16} + \frac{11}{16} - 1$?

35. What is the sum of $\frac{9}{15} + \frac{12}{15} + \frac{13}{15} - \frac{16}{15}$?

36. What is the sum of $\frac{20}{35} + \frac{18}{35} + \frac{16}{35} - 1$?

37. What is the sum of $\frac{29}{57} + \frac{11}{57} + \frac{19}{57} + \frac{16}{57}$?

38. What is the sum of $\frac{9}{100} + \frac{25}{100} + \frac{13}{100} + \frac{8}{100}$?

39. What is the difference between $\frac{42}{69}$ and $\frac{23}{69}$?

40. What is the difference between $\frac{85}{20}$ and $\frac{65}{20}$?

41. What is the difference between $2\frac{11}{13}$ and $1\frac{12}{13}$?

42. What is the sum of $\frac{20}{23} + \frac{21}{23} + 2$?

43. What is the difference between $4\frac{8}{13}$ and $3\frac{10}{13}$?

44. What is the sum of $1\frac{4}{11} + 1\frac{5}{11} + \frac{3}{11} + 2$?

45. What is the difference between $2\frac{7}{15}$ and $\frac{13}{15}$?

46. When are fractions said to have a common denominator?

47. How do we add or subtract fractions that have a common denominator?

48. Is there any difference between the manner of adding or subtracting fractions with a common denominator, and the adding or subtracting whole numbers of the *same kind?*

49. Why cannot fractions having different denominations be added in that state?

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SECTION XXVI.

DIVISION OF ONE FRACTION BY ANOTHER, WHEN THEY HAVE A COMMON DENOMINATOR.

WE divide one fraction by another, when they are of the same denomination, by dividing the numerator of the dividend by the numerator of the divisor, in the same manner as we divide two whole numbers.

1. How many times is $\frac{1}{2}$ contained in $\frac{2}{2}$? Ans. 2 times. Because $\frac{1}{2}$ can be subtracted from $\frac{2}{2}$ two times; $\frac{2}{2} - \frac{1}{2} = \frac{1}{2}$; $\frac{1}{2} - \frac{1}{2} = 0$.

2. How many times are $\frac{2}{8}$ contained in $\frac{4}{8}$?

3. How many times are $\frac{2}{8}$ contained in $\frac{6}{8}$?

4. How many times are $\frac{4}{12}$ contained in $\frac{9}{12}$?

5. How many times are $\frac{2}{5}$ contained in $\frac{5}{5}$? Ans. $2\frac{1}{2}$ times. Because $\frac{2}{5}$ can be subtracted from $\frac{5}{5}$ two and a half times; $\frac{5}{5} - \frac{2}{5} = \frac{3}{5}$; $\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$; as there is $\frac{1}{2}$ of two fifths remaining, it can be subtracted $2\frac{1}{2}$ times.

6. How many times are $\frac{3}{7}$ contained in $\frac{7}{7}$?

7. How many times are $\frac{3}{8}$ contained in $2\frac{5}{8}$? SOLUTION. $2\frac{5}{8}$ are equal to $\frac{21}{8}$; $\frac{3}{8}$ are contained in $\frac{21}{8}$ seven times. Ans. 7 times.

8. How many times are $\frac{7}{8}$ contained in $5\frac{3}{8}$?

9. How many times are $\frac{4}{9}$ contained in $7\frac{1}{9}$?

10. How many times are $\frac{6}{11}$ contained in $5\frac{5}{11}$?

11. How many times are $\frac{7}{12}$ contained in $11\frac{1}{2}$?

12. How many times are $1\frac{1}{4}$ contained in $7\frac{2}{4}$? SOLUTION. $1\frac{1}{4}$ are equal to $\frac{5}{4}$; $7\frac{2}{4}$ are equal to $\frac{30}{4}$: $\frac{5}{4}$ are contained in $\frac{30}{4}$ six times. Ans. 6 times.

13. A person expends $6\frac{3}{4}$ dollars for potatoes, paying $\frac{3}{4}$ of a dollar per bushel; how many bushels does he buy?

14. A gentleman distributed $3\frac{2}{7}$ dollars among some poor people, giving $\frac{4}{7}$ of a dollar to each person; how many persons shared in the charity?

15. How much calico can be purchased for $3\frac{5}{8}$ dollars, if $\frac{3}{8}$ of a dollar is paid for each yard?

16. If a lady pays $1\frac{3}{8}$ dollars for a yard of silk, how much can she purchase for $6\frac{7}{8}$ dollars?

17. A fruiterer wishes to put $17\frac{1}{2}$ bushels of apples in barrels; how many barrels will he require, providing each barrel will hold $2\frac{1}{2}$ bushels?

18. How many sacks, holding $1\frac{3}{8}$ bushels each, will be required to contain $11\frac{5}{8}$ bushels of potatoes?

19. A laborer was paid $10\frac{3}{8}$ dollars for performing a piece of work; how many days did it take him to do it, if he received $1\frac{1}{8}$ dollars for each day he labored?

20. If a horse will eat $2\frac{1}{7}$ tons of hay during the winter, how many horses, at that rate, will eat $6\frac{3}{7}$ tons in the same time?

21. A drover increases his flock by as many sheep as he can purchase for $19\frac{3}{4}$ dollars, paying $2\frac{3}{4}$ dollars a head; how many does he add to the drove?

22. How many barrels of flour can a trader purchase for $29\frac{1}{8}$ dollars, if he pays $7\frac{1}{8}$ dollars per barrel?

23. Paid $15\frac{3}{8}$ dollars for broadcloth, at the rate of $5\frac{1}{8}$ dollars a yard; how many yards did I purchase?

24. If a family of 7 persons consume a barrel of flour in $7\frac{1}{7}$ weeks, how many barrels will they consume in $21\frac{3}{7}$ weeks?

25. What number of men will it require to perform in $9\frac{3}{8}$ days a piece of work that one man can perform in $37\frac{1}{8}$ days?

26. How many times are $3\frac{1}{3}$ contained in $8\frac{1}{3}$?

27. How many times are $3\frac{1}{4}$ contained in $8\frac{1}{4}$?

28. How many times are $3\frac{1}{5}$ contained in $8\frac{1}{5}$?

29. How many times are $3\frac{1}{6}$ contained in $8\frac{1}{6}$?

30. How many times are $3\frac{1}{7}$ contained in $8\frac{1}{7}$?

31. How many times are $3\frac{1}{8}$ contained in $8\frac{1}{8}$?

32. How many times are $8\frac{1}{9}$ contained in $8\frac{1}{9}$?

33. How many times are $3\frac{1}{10}$ contained in $8\frac{1}{10}$?

34. How many times are $3\frac{1}{11}$ contained in $8\frac{1}{11}$?

35. How many times are $3\frac{1}{12}$ contained in $8\frac{1}{12}$?

36. How do we divide one fraction by another, when the fractions are of the same denomination?

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SECTION XXVII.

MULTIPLICATION AND DIVISION OF FRACTIONS BY A WHOLE NUMBER.

WE multiply a fraction by a whole number when we multiply the numerator by the whole number.

We divide a fraction by a whole number when we divide the numerator by the whole number.

Any fraction multiplied by its denominator gives the numerator for the whole number.

The only difference between the multiplication of fractions and the multiplication of units by whole numbers is, that in the former case the results are fractional parts, and in the latter whole numbers.

1. What do you understand by 3 times $\frac{2}{5}$? ANS. That $\frac{2}{5}$ are taken 3 times; thus, $\frac{2}{5} + \frac{2}{5} + \frac{2}{5}$.

2. How do you find the third of $\frac{6}{5}$? ANS. I divide $\frac{6}{5}$ into three equal parts, by dividing the numerator 6 by 3.

3. What is the third of $\frac{6}{5}$? ANS. $\frac{2}{5}$; because $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5}$.

4. What is the product of $\frac{3}{9} \times 6$?

5. What is the quotient of $\frac{18}{2} \div 6$?

6. What is the quotient of $\frac{15}{2} \div 5$?

7. What is the product of $\frac{7}{7} \times 3$?

8. What is the quotient of $\frac{19}{3} \div 3$?

9. What is the quotient of $\frac{36}{4} \div 6$?

10. What is the product of $\frac{2}{3} \times 5$?

11. What is the product of $\frac{7}{4} \times 6$?

12. How many oranges should I require, to be able to give $\frac{2}{3}$ of an orange to each of 8 boys?

13. How many pints of chestnuts should I require, to be able to give $\frac{5}{6}$ of a pint to each of 7 boys?

14. If a pair of shoes cost $\frac{7}{4}$ of a dollar, what fractional part of a dollar will 5 pairs cost? how many dollars?

15. If a man spend $\frac{7}{4}$ of a dollar each day, how much will he spend in 7 days? how much in 14 days?

16. A man owns $\frac{3}{12}$ of a ship, which he divides equally among his 3 children; what part of the ship does each child receive?

17. A gentleman distributed $\frac{40}{10}$ of a loaf of bread equally among 10 poor persons; how much did each person receive?

18. A lady spends $\frac{32}{4}$ of a dollar for 4 yards of silk; what fraction of a dollar does she pay for each yard? how many dollars?

19. Roland paid $\frac{33}{11}$ of a cent for 11 apples; what fraction of a cent, and how many cents, did he pay for each apple?

20. Eliot paid $\frac{25}{5}$ of a cent for 5 lemons; what did he pay for each lemon?

21. John bought 7 oranges for $\frac{84}{6}$ of a cent; what was the rate for an orange?

22. If it costs $6\frac{1}{4}$ cents per mile to ride in a stage-coach, what will it cost to ride 12 miles?

Obs. Multiply the whole number and the fraction separately.

23. If I pay $7\frac{2}{7}$ dollars for a barrel of flour, how much must I pay for 10 barrels at the same rate?

24. A store-keeper purchases 9 bags of Java coffee, for which he pays $11\frac{3}{8}$ dollars per bag; how much does he pay for the whole?

25. What would a package containing 12 pounds of coffee cost, at 12¼ cents per pound?

26. Bought 12 pounds of crushed sugar, at the rate of 9⅝ cents a pound; what was the cost of the sugar?

27. What is the cost of 15 pounds of brown Havana sugar, at the rate of 7¾ cents a pound?

28. A grocer sells rice for 5⅝ cents a pound; what will 16 pounds cost, at that rate?

29. James divided 2⅖ apples equally among 3 of his companions; what part of an apple did each boy receive? SOLUTION. Reduce 2⅖ to fifths, which are ¹²⁄₅; if 3 boys share ¹²⁄₅ of an apple equally between them, one boy will have the third of ¹²⁄₅, or ⅘ of an apple. ANS. ⅘ of an apple.

80. In a field there are 6⅘ acres of land, which a gentleman wishes to divide into 4 equal lots; how much land will each lot contain?

31. A farmer sold 5 bushels of oats for 2½ dollars; what fractional part of a dollar did he receive for each bushel?

32. If 5 bushels of potatoes are sold for 3⅛ dollars, what is that per bushel?

33. Bought 10 bushels of apples for 11⅝ dollars; what is the cost per bushel, at that rate?

34. A market-man sold melons at the rate of 12 for 14⅔ shillings; what did they cost apiece?

35. A gardener raised 13 bushels of peas, which he sold for 14⅝ dollars; what was the price per bushel, at that rate?

36. If 4 boxes of butter are given in exchange for 8 bushels of apples that are worth 1¼ dollars per bushel, what is the value of a box of the butter?

87. If 21 pounds of Oolong tea are given in exchange for 7 drums of figs that are worth 1⅔ dollars a drum, what is the cost per pound for the tea?

38. How do we multiply a fraction by a whole number?

39. How do we divide a fraction by a whole number?

40. What difference is there between the multiplication of fractions and the multiplication of units by whole numbers?

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SECTION XXVIII.

MULTIPLICATION AND DIVISION OF FRACTIONS BY A WHOLE NUMBER.

We also multiply a fraction by a whole number when we divide the denominator by the whole number.

We also divide a fraction by a whole number when we multiply the denominator by the whole number.

1. What is the product of $\frac{3}{12} \times 6$? SOLUTION. $\frac{3}{12} \times 6 = \frac{3}{2}$, the denominator 12 being divided by 6; $\frac{3}{2} = 1\frac{1}{2}$. ANS. $1\frac{1}{2}$.

2. What is the product of $\frac{7}{6} \times 3$? ANS. $\frac{7}{2}$, or $3\frac{1}{2}$.

3. What is the product of $\frac{3}{15} \times 5$?

4. What is the product of $\frac{36}{9} \times 3$?

5. What is the product of $\frac{18}{2} \times 4$?

6. What is the product of $\frac{12}{2} \times 9$?

7. What is the quotient of $\frac{8}{3} \div 3$? SOLUTION. $\frac{8}{3} \div 3 = \frac{8}{27}$, the denominator 9 being multiplied by 3; $\frac{6}{27} = \frac{2}{9}$. ANS. $\frac{2}{9}$.

8. What is the quotient of $\frac{18}{3} \div 3$? ANS. $\frac{18}{9}$, or 2.

9. What is the quotient of $\frac{18}{2} \div 6$?

10. What is the quotient of $\frac{15}{3} \div 5$?

11. What is the quotient of $\frac{33}{3} \div 6$?

12. What is the quotient of $\frac{34}{2} \div 7$?

13. What is the quotient of $\frac{28}{9} \div 5$?

14. What is the quotient of $\frac{38}{11} \div 7$?

15. What is the quotient of $\frac{18}{13} \div 6$?

16. What is the quotient of $\frac{75}{17} \div 7$?

17. What is the product of $\frac{7}{18} \times 6$?

18. What is the product of $\frac{100}{20} \times 5$?

19. What is the quotient of $\frac{100}{20} \div 5$?

20. What is the quotient of $\frac{135}{5} \div 5$?

21. If 8 yards of checked matting, $\frac{3}{4}$ wide, cost $2\frac{1}{4}$ dollars, what is it per yard? SOLUTION. In $2\frac{1}{4}$ there are $\frac{9}{4}$; if 8 yards cost $\frac{9}{4}$ of a dollar, 1 yard will cost $\frac{1}{8}$ of $\frac{9}{4}$ of a dollar, which is $\frac{9}{32}$ of a dollar. ANS. $\frac{9}{32}$ of a dollar.

22. If 8 yards of checked matting cost $2\frac{1}{4}$ dollars, what will 2 yards cost?

23. If 10 yards of twilled Venetian stair-carpeting cost $9\frac{1}{8}$ dollars, what is the cost of 1 yard? what is the cost of 6 yards?

24. If 6 yards of 3 ply carpeting cost $7\frac{1}{2}$ dollars, what is the cost of 1 yard? what is the cost of 21 yards?

25. Bought 8 yards of linen damask for $5\frac{6}{10}$ dollars; what was the cost of 1 yard? what would 16 yards cost, at that rate?

26. A lady purchased 7 yards of linen for $6\frac{1}{4}$ dollars; what would 8 yards cost, at that rate?

27. A piece of cotton cloth, containing 25 yards, was sold for $4\frac{1}{8}$ dollars; how much was that a yard? what was the cost of 8 yards?

28. If 24 linen doylies cost $3\frac{1}{6}$ dollars, what is the cost of each one? what is the cost of 12 of them? what is the cost of 18?

29. In what two ways can we multiply a fraction by a whole number?

30. In what two ways can we divide a fraction by a whole number?

31. When we multiply a fraction by a whole number, is the product larger or smaller than the multiplicand?

32. When we divide a fraction by a whole number, is the quotient larger or smaller than the dividend?

33. How is a fraction made greater? See Sect. XIX.

34. How is a fraction made smaller?

OBS. When the multiplication or division of fractions by whole numbers occurs in the succeeding sections, the pupil can make use of either of the above methods, as is most convenient to him.

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SECTION XXIX.

MULTIPLICATION OF A WHOLE NUMBER BY A FRACTION.

WE multiply by a fraction when we take the part or parts of the multiplicand indicated by the fractional multiplier. Thus, we multiply 8 by $\frac{3}{4}$ when we take 3 times the fourth ($\frac{1}{4}$) of 8.

1. Multiply 3 by $\frac{1}{2}$. SOLUTION. The half of 1 is $\frac{1}{2}$, and the half of 3 is $\frac{3}{2}$. ANS. $\frac{3}{2}$.

2. What are 3 times the half of 5? SOLUTION. The half of 5 is $\frac{5}{2}$ of 1, and 3 times $\frac{5}{2}$ are $\frac{15}{2}$, or $7\frac{1}{2}$. ANS. $7\frac{1}{2}$.

3. Give 3 times the half of 5 in another form. ANS. $3 \times \frac{5}{2}$, or $\frac{3}{2}$ of 5.

4. Multiply 6 by $\frac{2}{3}$.

5. Multiply 7 by $\frac{3}{4}$.

6. How do you multiply 7 by $\frac{3}{4}$? ANS. I take 3 times the fourth of 7.

7. What is the product of $8 \times \frac{3}{8}$ (8 multiplied by $\frac{3}{8}$)? of $8 \times \frac{5}{8}$? of $8 \times \frac{7}{8}$?

8. What is the product of $10 \times \frac{4}{5}$? of $10 \times \frac{3}{5}$? of $10 \times \frac{2}{5}$?

9. What is the product of 9 multiplied by $\frac{5}{9}$? by $\frac{7}{9}$? by $\frac{8}{9}$? by $\frac{11}{9}$?

10. What is the product of $12 \times \frac{3}{8}$? of $12 \times \frac{5}{8}$? of $12 \times \frac{7}{8}$? of $12 \times \frac{10}{8}$?

11. How do you multiply 12 by $\frac{7}{8}$?

12. A man who had found a purse containing 13 dollars paid $\frac{3}{10}$ of the money for advertising it; how much did he pay for advertising it?

13. At 20 dollars a ton, what is the cost of $\frac{2}{5}$ of a ton of hay? what is the cost of $\frac{8}{9}$ of a ton?

14. What is the product of $14 \times \frac{1}{2}$? of $14 \times \frac{1}{7}$?

15. Cocoa from Caraccas is sold for 9 cents a pound; what is the cost of $\frac{2}{13}$ of a pound?

16. Multiply 9 by $\frac{3}{13}$; 9 by $\frac{6}{13}$; 9 by $\frac{8}{13}$.

17. A man bought Java coffee at 11 cents a pound; how much did he pay for $\frac{5}{9}$ of a pound? how much for $\frac{7}{9}$ of a pound? how much for $\frac{12}{9}$ of a pound?

18. What is the product of $11 \times \frac{4}{9}$? of $11 \times \frac{6}{9}$? of $11 \times \frac{8}{9}$? of $11 \times \frac{10}{9}$?

19. What will cost $1\frac{3}{8}$ barrels of Genesee flour, at 5 dollars a barrel?

20. Multiply 5 by $1\frac{7}{8}$ ($\frac{15}{8}$).

21. Richmond City flour is worth 8 dollars a barrel; what is the cost of $1\frac{3}{11}$ barrels, at that rate?

22. Chinese glue is worth 23 cents per pound; what is the value of $\frac{9}{16}$ of a pound, at that rate?

23. If I buy American glue for 8 cents a pound, what do I pay for $\frac{9}{16}$ of a pound?

24. What is the product of $8 \times 1\frac{4}{11}$? of $23 \times \frac{9}{16}$? of $8 \times \frac{11}{16}$?

25. Bought hops for 20 cents a pound; how much did I pay for $\frac{5}{14}$ of a pound?

26. What is the cost of a Calcutta hide, weighing $9\frac{3}{4}$ pounds, at 14 cents a pound? First find the cost of 9 pounds, and then of $\frac{3}{4}$ of a pound.

27. What are $6\frac{3}{4}$ times 18? $5\frac{3}{4}$ times 18?

28. If a man earn 30 cents by working an hour, how much will he earn in $3\frac{4}{4}$ hours?

29. When do we multiply by a fraction?

30. Give an example of the multiplication of a whole number by a fraction.

SECTION XXX.

REDUCTION OF COMPOUND FRACTIONS, OR MULTIPLICA-
TION OF A FRACTION BY A FRACTION.

WE multiply a fraction by a fraction when we multiply the numerators by one another to give the numerator of the product, and the denominators by one another to give the denominator of the product. Hence, the fraction of a fraction is the same thing as the multiplication of a fraction by a fraction.

Thus, $\frac{2}{3}$ multiplied by $\frac{4}{5}$ is an operation whereby we take $\frac{2}{3}$ four fifths of once.

A *compound fraction* is a fraction of a fraction, or any number of fractions connected by the word *of*. Thus $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{4}{5}$.

1. If I divide an orange into 3 equal parts, what part of the whole orange is one of the parts? If this third be divided into 2 equal parts, what part of $\frac{1}{3}$ will 1 of the parts be? ANS. $\frac{1}{2}$ of $\frac{1}{3}$.

2. What part of a whole orange is $\frac{1}{2}$ of $\frac{1}{3}$ of an orange? ANS. $\frac{1}{6}$. Because each third contains 2 equal parts, and the whole is therefore divided into 6 equal parts.

3. What does this prove? ANS. That $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$.

4. What is the half of a half? ANS. $\frac{1}{4}$. Because, when a half is divided into 2 equal parts, 1 part is a fourth of the whole.

5. What is the product of $\frac{1}{4} \times \frac{1}{2}$? SOLUTION. To take $\frac{1}{4}$ one half times is the same as $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$. ANS. $\frac{1}{8}$.

6. What is $\frac{1}{2}$ of $\frac{1}{7}$? $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{1}{9}$? $\frac{1}{2}$ of $\frac{1}{10}$? $\frac{1}{2}$ of $\frac{1}{11}$? $\frac{1}{2}$ of $\frac{1}{12}$?

7. What is $\frac{1}{3}$ of $\frac{1}{3}$? Why? Because, if $\frac{1}{3}$ be divided into 3 equal parts, there will be 9 of such parts in a whole one.

8. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{3}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{3}$ of $\frac{1}{7}$? $\frac{1}{3}$ of $\frac{1}{8}$? $\frac{1}{3}$ of $\frac{1}{9}$? $\frac{1}{3}$ of $\frac{1}{10}$? $\frac{1}{3}$ of $\frac{1}{11}$? $\frac{1}{3}$ of $\frac{1}{12}$?

9. If an orange be divided into 3 equal parts, into how many pieces must I cut one of the parts, in order to obtain twelfths of the orange?

10. A boy gave $\frac{1}{3}$ of an orange to one companion, and $\frac{1}{4}$ of the remainder to another; what part of the orange did the last boy receive?

11. What is $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{4}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{4}$ of $\frac{1}{6}$? $\frac{1}{4}$ of $\frac{1}{7}$? $\frac{1}{4}$ of $\frac{1}{8}$? $\frac{1}{4}$ of $\frac{1}{9}$? $\frac{1}{4}$ of $\frac{1}{10}$? $\frac{1}{4}$ of $\frac{1}{11}$? $\frac{1}{4}$ of $\frac{1}{12}$?

12. In a rod there are $16\frac{1}{2}$ feet, or $\frac{33}{2}$ of a foot; what number of feet is there in $\frac{1}{6}$ of a rod?

13. What is $\frac{1}{5}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{4}$? $\frac{1}{5}$ of $\frac{1}{5}$? $\frac{1}{5}$ of $\frac{1}{6}$? $\frac{1}{5}$ of $\frac{1}{7}$? $\frac{1}{5}$ of $\frac{1}{8}$? $\frac{1}{5}$ of $\frac{1}{9}$? $\frac{1}{5}$ of $\frac{1}{10}$? $\frac{1}{5}$ of $\frac{1}{11}$? $\frac{1}{5}$ of $\frac{1}{12}$?

14. In one rod there are $5\frac{1}{2}$ yards; what fraction of a yard is there in $\frac{1}{6}$ of a rod?

15. What is $\frac{1}{6}$ of $\frac{1}{4}$? $\frac{1}{6}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{8}$? $\frac{1}{6}$ of $\frac{1}{11}$? $\frac{1}{6}$ of $\frac{1}{13}$? $\frac{1}{6}$ of $\frac{1}{15}$? $\frac{1}{6}$ of $\frac{1}{17}$? $\frac{1}{6}$ of $\frac{1}{19}$?

16. What is $\frac{1}{7}$ of $\frac{1}{7}$? $\frac{1}{7}$ of $\frac{1}{8}$? $\frac{1}{10}$ of $\frac{1}{11}$? $\frac{1}{11}$ of $\frac{1}{10}$? $\frac{1}{12}$ of $\frac{1}{11}$? $\frac{1}{11}$ of $\frac{1}{12}$?

17. A mill is $\frac{1}{10}$ of a cent, a cent is $\frac{1}{10}$ of a dime, and a dime is $\frac{1}{10}$ of a dollar; what fraction of a dollar is 1 mill?

18. What is $\frac{1}{10}$ of $\frac{1}{10}$ of $\frac{1}{10}$?

19. Henry had $\frac{1}{3}$ of a dollar, and gave his brother $\frac{2}{7}$ of his money; what fraction of a dollar did his brother receive? SOLUTION. $\frac{1}{7}$ of $\frac{1}{3}$ of a dollar is $\frac{1}{21}$ of a dollar; $\frac{2}{7}$ of $\frac{1}{3}$ of a dollar are 2 times $\frac{1}{21}$, or $\frac{2}{21}$ of a dollar. ANS. $\frac{2}{21}$ of a dollar.

20. What is the product of $\frac{1}{3} \times \frac{3}{7}$? $\frac{1}{3} \times \frac{4}{7}$?

21. What is the product of $\frac{1}{4} \times \frac{2}{5}$? $\frac{1}{4} \times \frac{7}{9}$?

22. An inch is $\frac{1}{12}$ of $\frac{1}{3}$ of a yard; what fraction of a yard are 8 inches?

23. One farthing is $\frac{1}{4}$ of a penny, and one penny is $\frac{1}{12}$ of a shilling; what part of a shilling are 3 farthings?

24. If a bushel of oats cost $\frac{5}{7}$ of a dollar, what are $\frac{4}{9}$ of a bushel worth? ANS. $\frac{20}{63}$ of a dollar. Because

$\frac{1}{8}$ of $\frac{5}{7}$ = $\frac{5}{56}$, and 4 times the ninth of $\frac{5}{7}$ = 4 times $\frac{5}{63}$, or $\frac{20}{63}$.

25. What are $\frac{4}{5}$ of $\frac{5}{7}$? $\frac{4}{5}$ of $\frac{5}{6}$? $\frac{6}{5}$ of $\frac{4}{5}$? $\frac{5}{6}$ of $\frac{2}{11}$? $\frac{9}{10}$ of $\frac{5}{12}$?

26. A merchant bought $\frac{5}{6}$ of a ship, and afterwards sold $\frac{3}{4}$ of his share; what part of the ship did he sell?

27. A merchant owning $\frac{7}{12}$ of a ship sold $\frac{5}{11}$ of his share; what part of the ship did he sell?

28. What is the product of $\frac{9}{2} \times 1\frac{2}{3}$, or $\frac{5}{3}$?

29. What is the product of $\frac{3}{4} \times 7\frac{2}{3}$?

30. If an apple cost $\frac{3}{4}$ of a cent, what is the cost of $9\frac{1}{3}$ apples, at that rate?

Obs. Reduce mixed numbers to improper fractions before multiplying.

31. What is the product of $\frac{2}{3} \times 9\frac{3}{4}$?

32. A fisherman, upon his return from the Grand Banks, disposes of his cod-fish for $3\frac{1}{3}$ dollars per quintal (112 pounds); what is the cost of $\frac{3}{4}$ of a quintal, at that rate? of $\frac{5}{8}$ of a quintal?

33. What is the product of $\frac{7}{2} \times \frac{3}{4}$? $\frac{7}{2} \times \frac{3}{8}$?

34. A grocer sells sperm oil for $1\frac{2}{5}$ dollars a gallon; what is the cost of $\frac{5}{6}$ of a gallon, at that rate?

35. What is the product of $\frac{7}{8} \times \frac{1}{2}$? $\frac{1}{2} \times \frac{7}{8}$?

36. No. 1 mackerel are worth $12\frac{1}{4}$ dollars a barrel; what is the price of $\frac{2}{3}$ of a barrel, at that rate?

37. What is the product of $\frac{2}{3} \times 1\frac{2}{5}$?

38. A grocer sells extra lard oil for $\frac{9}{10}$ of a dollar per gallon; what is the cost of $\frac{3}{4}$ of a gallon, at that rate?

39. What is the product of $\frac{5}{8} \times 1\frac{3}{8}$?

40. Bought linseed oil for $\frac{7}{10}$ of a dollar a gallon; how much did I pay for $5\frac{1}{8}$ gallons?

41. What is the product of $1\frac{2}{3} \times \frac{3}{4}$?

42. Lubec herring are sold for $\frac{1}{2}$ dollar a box; what is the cost of $9\frac{1}{7}$ boxes, at that price?

43. What is $\frac{5}{7}$ of $\frac{4}{5}$? $\frac{5}{9}$ of $\frac{2}{3}$?

44. Sperm candles are worth $\frac{3}{10}$ of a dollar a pound; what is the value of $5\frac{7}{8}$ pounds, at that price?

45. If No. 2 mackerel are worth $10\frac{7}{8}$ dollars a barrel, what is the price of $\frac{2}{3}$ of a barrel, at that rate?

46. Mould candles are worth $\frac{1}{8}$ of a dollar a pound; what is the value of $12\frac{5}{8}$ pounds, at that price?

47. Bought adamantine candles, paying $\frac{7}{25}$ of a dollar for each pound of candles bought; what did I pay for a box containing $10\frac{1}{2}$ pounds of the candles?

48. If an orange cost $1\frac{1}{3}$ cents, what is the cost of $6\frac{1}{4}$ oranges, at that rate?

49. What is $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$?

50. What is the product of $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{5}$?

51. What is the product of $\frac{1}{4} \times \frac{4}{5} \times \frac{5}{6}$?

52. What is the product of $\frac{2}{3} \times \frac{3}{8} \times \frac{4}{7}$?

53. What is the product of $\frac{5}{6} \times \frac{3}{7} \times \frac{3}{8}$?

Obs. The answers to the last few questions may be given without explanation.

54. What is a compound fraction?

55. Is there any difference in the operation between the reduction of compound fractions and the multiplication of a fraction by a fraction?

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SECTION XXXI.

REDUCTION OF FRACTIONS TO A COMMON DENOMINATOR.

WE bring two fractions to a *common* denominator, that is, to the same denominator, by multiplying both numerator and denominator of each fraction by the denominator of the other.

More than two fractions are brought to a common denominator by multiplying the numerator and denominator of each by the denominators of all the others.

The least common multiple of the denominators is the *least* common denominator.

If an apple be divided into 3 equal parts, each part is called a third ($\frac{1}{3}$); and if each third be cut into 2 equal parts, the whole apple is cut into 6 equal parts, and each part is called a sixth ($\frac{1}{6}$) of the whole apple; 2 parts ($\frac{2}{6}$) are the same as a third ($\frac{1}{3}$).

1. In $\frac{1}{2}$ how many fourths are there? Ans. $\frac{2}{4}$. In 1 there are $\frac{4}{4}$, and one half of $\frac{4}{4}$ is $\frac{2}{4}$.

2. To what denominator must I bring $\frac{1}{2}$ and $\frac{1}{3}$, in order that I may add them together?

3. In $\frac{1}{3}$ how many ninths are there? how many sixths?

4. How many twelfths are there in $\frac{1}{3}$? in $\frac{1}{4}$? in $\frac{1}{6}$?

5. To what denominator must I bring $\frac{1}{2}$ and $\frac{1}{3}$ in order to add them together? $\frac{1}{2}$ and $\frac{1}{4}$? $\frac{1}{2}$ and $\frac{1}{6}$? $\frac{1}{3}$ and $\frac{1}{6}$? $\frac{1}{4}$ and $\frac{1}{6}$?

6. How many fifteenths in $\frac{1}{3}$? in $\frac{1}{5}$? in $\frac{1}{3} + \frac{1}{5}$?

7. Which is the larger, $\frac{1}{3}$ or $\frac{1}{6}$? how much?

8. Reduce $\frac{1}{3}$ and $\frac{1}{6}$ to fractions having the same denominator.

9. How many eighteenths in $\frac{1}{2}$? in $\frac{1}{3}$? in $\frac{1}{9}$? in $\frac{1}{2} + \frac{1}{6}$? in $\frac{1}{2} + \frac{1}{3} + \frac{1}{9}$? in $\frac{1}{2} - \frac{1}{6} + \frac{1}{9}$?

10. Reduce $\frac{2}{3}$ and $\frac{4}{6}$ to fractions having the same denominator.

11. Reduce $\frac{2}{3}$ and $\frac{4}{5}$ to a common denominator.

12. Reduce $\frac{1}{3}$ and $\frac{1}{7}$ to a common denominator.

13. How many twenty-firsts in $\frac{1}{2}$? in $\frac{2}{3}$? in $\frac{1}{7}$? in $\frac{2}{7}$? in $\frac{3}{7}$? in $\frac{4}{7}$? in $\frac{5}{7}$? in $\frac{6}{7}$?

14. What is the sum of $\frac{1}{3}$ and $\frac{1}{7}$?

15. What is the sum of $\frac{2}{3}$ and $\frac{1}{7}$? $\frac{2}{3}$ and $\frac{2}{7}$? $\frac{2}{3}$ and $\frac{4}{7}$?

16. What is the difference between $\frac{2}{3}$ and $\frac{2}{7}$? $\frac{2}{3}$ and $\frac{4}{7}$?

17. Which is the greater fraction, $\frac{2}{3}$ or $\frac{5}{7}$? how much?

18. Reduce to a common denominator $\frac{1}{2}$ and $\frac{1}{8}$; $\frac{1}{3}$ and $\frac{2}{9}$; $\frac{1}{2}$ and $\frac{3}{8}$; $\frac{1}{2}$ and $\frac{1}{7}$.

19. What is the sum of $\frac{1}{3}$ plus $\frac{3}{8}$? $\frac{1}{3}$ plus $\frac{7}{8}$?

20. What is the sum of $\frac{3}{8} + \frac{5}{9}$? $\frac{3}{8} + \frac{6}{8}$? $\frac{3}{8} + \frac{7}{8}$?

21. What is the difference between $\frac{2}{8}$ and $\frac{5}{8}$? $\frac{7}{8}$ and $\frac{5}{8}$?

22. How can we add fifths and ninths?

23. What is the sum of $\frac{1}{2}$ and $\frac{4}{5}$? $\frac{2}{3}$ and $\frac{5}{8}$? $\frac{4}{5}$ and $\frac{7}{9}$?

24. What is the difference between $\frac{1}{2}$ and $\frac{1}{7}$? $\frac{2}{3}$ and $\frac{1}{9}$? $\frac{3}{4}$ and $\frac{2}{9}$? $\frac{4}{5}$ and $\frac{5}{8}$? $\frac{4}{5}$ and $\frac{6}{8}$? $\frac{7}{9}$ and $\frac{2}{3}$?

25. How can we add fourths and fifths?

26. To what denominator must I bring halves, thirds and fourths, in order that I may add them together?

27. What is the sum of $\frac{4}{5}$ and $\frac{5}{6}$?

28. What is the sum of $\frac{3}{4}$ and $\frac{4}{4}$?

29. What is the difference between $\frac{3}{4}$ and $\frac{4}{4}$?

30. What is the common denominator of sevenths and eighths? of eighths and ninths? of ninths and tenths? of tenths and elevenths?

31. What is the sum of $\frac{5}{9}$ and $\frac{7}{4}$?

32. What is the sum of $1\frac{1}{4}$ and $1\frac{2}{8}$?

33. What is the difference between $1\frac{6}{8}$ and $1\frac{1}{4}$?

34. What is the difference between $2\frac{1}{2}$ and $1\frac{7}{8}$?

35. Three boys share a pine-apple in the following manner: the first boy takes $\frac{3}{8}$ of it, the second $\frac{2}{6}$ of it, and the third boy has the remainder; what part of the pine-apple does the third boy get?

36. Subtract $\frac{5}{8}$ plus $\frac{6}{8}$ from 2.

37. A merchant who owned $\frac{7}{9}$ of a ship sold $\frac{3}{4}$ of her; what part of the ship did he still own?

38. If $1\frac{1}{4}$ bushels of apples be taken from a barrel containing $2\frac{3}{16}$ bushels, what part of a bushel will remain in the barrel?

39. A, B, C and D, own a ship together; A owns $\frac{1}{5}$ of her, B owns $\frac{1}{3}$ of her, C owns $\frac{1}{4}$ of her, and D owns the remainder; what part of the ship belongs to D?

40. Charles has $\frac{3}{4}$ of a dollar, and he finds $\frac{5}{12}$ of a

dollar more; what fraction of a dollar is required to make his money amount to a dollar?

41. Edwin has an apple and an orange of equal size; he divides the apple into 7 equal parts, and the orange into 9 equal parts, and gives away 3 parts of the apple and 4 parts of the orange; what part of each has he remaining? how much more of one than of the other?

42. The boys in a class are arranged in four divisions; the first division contains $\frac{1}{4}$ of the whole, the second $\frac{3}{14}$ of the whole, the third $\frac{1}{7} + \frac{3}{28}$ of the whole, and the fourth contains the remainder; what part of the class does the fourth division contain?

43. Theodore paid $\frac{1}{2}$ of a dollar for his Reader, $\frac{1}{8} + \frac{1}{4}$ of a dollar for his Geography and Atlas, $\frac{1}{6}$ of a dollar for his Slate, and $\frac{3}{20}$ of a dollar for his Mental Arithmetic; what fraction of a dollar did he pay for the whole? He gave in payment a $2\frac{1}{4}$ dollar gold-piece; how much money did he receive back?

44. A man can reap $\frac{5}{7}$ of a field of wheat in a day, his elder son can reap $\frac{2}{3}$ of it in a day, and his younger son $\frac{3}{8}$ of it; what part of the field can the two sons together reap in a day? how much more than the father alone?

45. $\frac{3}{8} + \frac{3}{8}$ — what fraction $= \frac{5}{7}$?

46. $\frac{5}{7} + \frac{3}{8}$ — what fraction $= \frac{2}{3}$?

47. To what denomination must we bring sixths and fourteenths, in order to add them together?

48. What is the least common denominator of sixths and fourteenths?

49. $\frac{5}{16} + \frac{3}{8} + \frac{3}{4} + \frac{1}{2} +$ what improper fraction $= 5$?

50. $\frac{1}{20} + \frac{6}{16} + \frac{3}{5} + \frac{2}{4} + \frac{1}{3}$ — what fraction $= 2$?

51. $\frac{1}{30} + \frac{2}{15} + \frac{3}{10} + \frac{5}{8} + \frac{2}{3} + \frac{1}{2}$ — what fraction $= 3$?

52. $3\frac{4}{7} + 6\frac{5}{8} +$ what fraction $= 10$?

53. How do we bring fractions to a common denominator?

54. When there are more than two fractions, how do we bring them to a common denominator ? ·

55. What is the *least* common denominator of two or more fractions ?

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SECTION XXXII.

DIVISION OF A WHOLE NUMBER BY A FRACTION, AND THE DIVISION OF A FRACTION BY A FRACTION.

WE divide a whole number by a fraction when we bring the whole number to an improper fraction of the same denomination, and divide its numerator by the numerator of the divisor.

OBS. The dividend is the same part of the quotient that the divisor is of a unit. Thus, $1 \div \frac{1}{4} = 4$, and 1 (the dividend) is $\frac{1}{4}$ of 4 (the quotient).

1. Divide 3 by $\frac{1}{4}$. SOLUTION. In 3 there are $\frac{12}{4}$, and 1 fourth is contained in 12 fourths 12 times. ANS. 12.

2. Divide 5 by $\frac{2}{3}$. SOLUTION. In 5 there are $\frac{15}{3}$, and 2 thirds are contained in 15 thirds $7\frac{1}{2}$ times. ANS $7\frac{1}{2}$.

3. How many yards of mousseline-de-laine can be purchased for 6 dollars, when the price is $\frac{4}{5}$ of a dollar a yard ?

4. Divide 11 by $\frac{2}{3}$; 12 by $\frac{3}{4}$; 13 by $\frac{4}{5}$; 12 by $\frac{7}{8}$; 10 by $\frac{5}{6}$.

5. At $\frac{3}{4}$ of a cent apiece, how many crackers can you buy for 15 cents ?

6. What is the quotient of $16 \div \frac{2}{3}$? of $18 \div \frac{4}{5}$?

7. How many bushels of potatoes, at $\frac{5}{8}$ of a dollar per bushel, can be bought for 10 dollars ?

8. 15 are how many times $\frac{3}{8}$? $\frac{4}{5}$? $\frac{5}{8}$?

9. Henry received from his father $\frac{3}{4}$ of a dollar a volume for reading the history of England, and, upon

counting his money, he found he had received 9 dollars; how many volumes had he read?

10. Albert earns $\frac{8}{10}$ of a dollar a week; how long will he be in earning enough to buy a suit of clothes which shall cost 12 dollars?

11. Divide 16 by $3\frac{1}{4}$. SOLUTION. In $3\frac{1}{4}$ there are $\frac{13}{4}$, and in 16 there are $\frac{64}{4}$; 13 fourths are contained in 64 fourths $4\frac{12}{13}$ times. ANS. $4\frac{12}{13}$.

12. If a class recite $6\frac{2}{5}$ pages in history in a day, how many days will it take them to recite a chapter containing 32 pages?

13. 25 are how many times $11\frac{1}{4}$? $15\frac{5}{8}$?

14. How many pounds of raisins, at $9\frac{2}{3}$ cents per pound, can be bought for 29 cents?

15. Divide 14 by $6\frac{2}{3}$.

16. Divide 18 by $8\frac{1}{3}$.

We divide a fraction by a fraction when we bring both to a common denominator, and divide the numerator of the dividend by the numerator of the divisor. Thus, to divide $\frac{2}{3}$ by $\frac{4}{7}$, we say $\frac{2}{3} = \frac{14}{21}$; and $\frac{4}{7} = \frac{12}{21}$; and $14 \div 12 = 1\frac{1}{6}$.

17. What is the quotient of $\frac{1}{2}$ divided by $\frac{4}{7}$?

18. At one dollar for $\frac{1}{2}$ of a day's work, how much will a man earn in $\frac{5}{6}$ of a day? how much in $\frac{3}{8}$ of a day? how much in $\frac{5}{7}$ of a day?

19. Divide $\frac{4}{5}$ by $\frac{1}{4}$; $\frac{5}{8}$ by $\frac{1}{4}$; $\frac{7}{9}$ by $\frac{1}{4}$.

20. How many hours must a man work to earn $\frac{7}{8}$ of a dollar, if he earn $\frac{2}{5}$ of a dollar an hour?

21. Divide $\frac{5}{6}$ by $\frac{3}{4}$; $\frac{6}{7}$ by $\frac{2}{5}$; $\frac{7}{8}$ by $\frac{3}{5}$.

22. Divide $\frac{4}{5}$ by $\frac{3}{8}$; $1\frac{1}{4}$ by $\frac{4}{7}$; $\frac{6}{7}$ by $\frac{5}{6}$.

When the pupil becomes familiar with the above process, he may be taught to invert the divisor, and multiply the dividend by it in that form. $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12}$, or $\frac{5}{6}$. It is explained thus: $\frac{1}{3}$ is contained in $\frac{4}{5}$, 3 times $\frac{4}{5}$, or $\frac{12}{5}$ times; and $\frac{2}{3}$ are contained in the same, $\frac{1}{2}$ of $\frac{12}{5}$, or $\frac{6}{5}$ times. (See Obs.) It must be fully shown to the pupil, that as many times as the *divisor* is

diminished, so many times the *quotient* is increased, and as many times as the *divisor* is increased, so many times the *quotient* is diminished, and that the *quotient* is increased or diminished as many times as the *dividend* is increased or diminished.

23. Divide $\frac{5}{7}$ by $\frac{9}{11}$; $\frac{5}{8}$ by $\frac{3}{4}$; $\frac{3}{7}$ by $\frac{5}{6}$.

24. At $\frac{5}{8}$ of a dollar a pair, how many pairs of gloves can I buy for $3\frac{1}{8}$ dollars? how many pairs for $2\frac{1}{2}$ dollars? how many for $4\frac{1}{8}$ dollars?

25. Divide $\frac{4}{5}$ by $\frac{7}{11}$; $\frac{3}{7}$ by $\frac{6}{11}$; $\frac{6}{11}$ by $\frac{3}{7}$.

26. What fraction, multiplied by $\frac{3}{10}$, will give the product $1\frac{4}{5}$? $2\frac{3}{4}$? $2\frac{4}{5}$?

27. My neighbor, Mr. Steady, mows in one day $1\frac{1}{4}$ acres of grass, for which he receives 1 dollar; how much must I pay him, at that rate, for mowing a field containing $9\frac{3}{4}$ acres?

28. How many times are $4\frac{3}{4}$ contained in $5\frac{5}{8}$? $4\frac{3}{5}$ in $5\frac{7}{10}$?

29. A man buys wood $5\frac{1}{4}$ feet in length; into how many pieces must he saw each stick, in order to burn it in a stove $1\frac{3}{4}$ feet in length?

30. Divide $8\frac{4}{5}$ by $1\frac{1}{3}$; $7\frac{7}{8}$ by $3\frac{1}{3}$ (change $\frac{1}{3}$ to $\frac{3}{9}$).

31. A lady purchases lace for $\frac{2}{3}$ of a dollar a yard; how much lace can she buy for $5\frac{2}{4}$ dollars?

32. Divide $5\frac{3}{4}$ by $\frac{2}{5}$; $5\frac{1}{4}$ by $\frac{3}{4}$.

33. A lady purchased alpaca for $\frac{5}{7}$ of a dollar a yard; how much alpaca could she buy for $6\frac{1}{2}$ dollars?

34. Divide $6\frac{1}{2}$ by $\frac{5}{7}$; $5\frac{1}{4}$ by $\frac{5}{6}$.

35. A charitable society distributed $6\frac{3}{4}$ dollars among several poor families, giving $2\frac{3}{8}$ dollars to each family; how many families shared the donation?

36. When do we divide a whole number by a fraction?

37. When do we divide a fraction by a fraction?

Obs. We bring both fractions to a common denominator, and divide one numerator by the other, when we invert the divisor and multiply a fraction by a fraction.

SECTION XXXIII.

1. ⅔ of 9 is one fourth of what number? SOLUTION. ⅔ of 9 are 6; and if 6 is one fourth of a certain number, four fourths are 4 times 6, or 24. ANS. 24.

2. ¾ of 12 is one sixth of what number?

3. ¾ of 12 are three sixths of what number?

4. ⅘ of 10 are four sevenths of what number

5. ⅝ of 18 are three eighths of what number?

6. ⅞ of 16 are seven ninths of what number?

7. ₁⁹₁ of 22 are six sevenths of what number?

8. ₁₁½ of 24 are two fifths of what number?

9. ⅝ of 81 are nine tenths of what number?

10. ⅔ of 21 are six elevenths of what number?

11. ₁⁷₁ of 44 are four fifths of what number?

12. ⅝ of 27 are six sevenths of what number?

13. ₃₃₂ of 75 are thirty-three forty-fifths of what number?

14. ₁₁⅞ of 100 are five sixths of what number?

15. ₁⁷₂ of 96 are fourteen fifteenths of what number?

16. ⅓ of 12 is one sixth of how many times 8? SOLUTION. ⅓ of 12 is 4, and four is one sixth of 6 × 4, or 24. 8 are contained in 24, 3 times. ANS. 3 times.

17. ⅔ of 9 is one fifth of how many times 5?

18. ¾ of 16 are two thirds of how many times 9?

19. ⅘ of 20 are four sevenths of how many times 8?

20. ⅝ of 18 are three sevenths of how many times 6?

21. ⅞ of 16 are seven ninths of how many times 6?

22. ⅝ of 28 are eight elevenths of how many times 3?

23. ⅝ of 24 are three eighths of how many times 5?

24. ⅞ of 27 are three eighths of how many times 7?

25. ⅘ of 63 are fifteen sixteenths of how many times 8?

26. ₃⁵₈ of 72 are ten elevenths of how many times 11?

27. $\frac{5}{8}$ of 72 are sixteen seventeenths of how many times 4 ?

28. $\frac{2}{3}$ of 24 are four fifths of how many times the eighth of 40 ? SOLUTION. $\frac{2}{3}$ of 24 are 16, and 16 are four fifths of 20. The eighth of 40 is 5, and five are contained in 20, 4 times. ANS. 4 times.

29. $\frac{3}{4}$ of 28 are two ninths of how many times the seventh of 42 ?

30. $\frac{3}{5}$ of 30 are nine elevenths of how many times the fifth of 55 ?

31. $\frac{7}{8}$ of 32 are two fifths of how many times the ninth of 63 ?

32. $\frac{5}{6}$ of 24 are four sevenths of how many times the seventh of 35 ?

33. $\frac{4}{9}$ of 45 are five elevenths of how many times the eighth of 56 ?

34. $\frac{5}{9}$ of 72 are eight elevenths of how many times the ninth of 36 ?

35. $\frac{4}{5}$ of 35 are five ninths of how many times the seventh of 63 ?

36. $\frac{9}{7}$ of 14 are six fifteenths of how many times the eleventh of 99 ?

37. $\frac{5}{6}$ of 54 are three sevenths of how many times the sixth of 48 ?

38. $\frac{5}{11}$ of 132 are twelve sevenths of how many times the ninth of 63 ?

39. $\frac{5}{9}$ of 72 are sixteen fifths of how many times the fourteenth of 70 ?

40. $\frac{4}{5}$ of 20 are eight thirds of what part of 42 ? SOLUTION. $\frac{4}{5}$ of 20 are 16, and 16 are eight thirds of 6, which is $\frac{1}{7}$ of 42. ANS. $\frac{1}{7}$.

41. $\frac{3}{4}$ of 56 are seven fourths of what part of 96 ?

42. $\frac{5}{12}$ of 132 are eleven fifteenths of what part of 30 ?

43. $\frac{8}{9}$ of 108 are sixteen fifths of what part of 45 ?

44. $\frac{4}{5}$ of 75 are five fourths of what part of 60 ?

45. $\frac{3}{4}$ of 21 are nine fourteenths of what part of 56?

46. $\frac{9}{13}$ of 52 are twelve elevenths of what part of 66?

47. $\frac{7}{17}$ of 24 are three fourths of what part of 70?

48. $\frac{2}{17}$ of 35 are fifteen sixteenths of what part of 100?

49. $\frac{1}{12}$ of 144 are six fourths of what part of 121?

50. $\frac{5}{9}$ of 63 are seven eighths of what part of 45?

———◆———

SECTION XXXIV.

1. A man had $4\frac{3}{4}$ dollars to distribute among some poor children; to how many children could he give $\frac{1}{8}$ of a dollar each?

2. A farmer sold ten small elm trees, at $\frac{5}{6}$ of a dollar for each tree; how many dollars did he get for them?

3. A drover spent $27\frac{1}{2}$ dollars for sheep at $2\frac{1}{2}$ dollars apiece, and afterwards sold the same sheep at $3\frac{1}{3}$ dollars apiece; how much did he gain by the transaction?

4. A lady bought ribbon for $2\frac{3}{4}$ dollars, lace for $3\frac{1}{4}$ dollars, and thread for $\frac{4}{5}$ of a dollar. She gave in payment a ten-dollar bill; how much money did she receive back?

5. How much must be added to the sum of $3\frac{1}{4}$ and $7\frac{3}{4}$ to make $15\frac{2}{3}$?

6. How many times does a wheel $5\frac{5}{6}$ feet in circumference turn round in going 100 feet?

7. A man has a field of $14\frac{2}{5}$ acres, which he mows at the rate of $2\frac{2}{5}$ acres a day; how many days does it take him to mow the field?

8. A grocer bought sugar at $5\frac{3}{4}$, $6\frac{3}{4}$, $7\frac{1}{4}$ and $8\frac{1}{4}$ cents a pound; what was the average price? What would be the price of 10 pounds of each of the first two kinds?

9. What must be added to 5 times $6\frac{4}{5}$ to make the sum $40\frac{1}{2}$?

10. James lives $\frac{3}{4}$ of a mile from his school, and Robert lives $\frac{2}{3}$ as far; how many rods does Robert walk in going to school, there being 320 rods in a mile?

11. Four boys shared a melon equally, and Joseph, one of the four, gave one-half of his share to his little brother; what part of the whole melon had Joseph left?

12. What must be subtracted from $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{10}$ to leave $\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{1}{2}$?

13. At $1\frac{1}{4}$ dollars a gallon for oil, what is the cost of a quart? What is the cost of 3 quarts? What is the cost of 2 gallons and 3 quarts?

14. One half of one fourth of a pole is in the mud, three fourths of one half of it is in the water, and the rest of it is above water; what part of the pole is above the water?

15. If you recite $2\frac{1}{2}$ pages of this book every day, how long will it take you to recite $42\frac{1}{2}$ pages?

16. A man bought some hay for 90 dollars, and sold it for $\frac{7}{6}$ of what it cost him; how much did he gain by the trade?

17. If I sell my goods so as to gain $\frac{3}{8}$ of the cost, at what part of the cost do I sell them? What did I pay for sugar which I sell for 11 cents a pound?

18. 25 is five sevenths of what number?

19. If to a certain number we add $\frac{2}{5}$ of itself, the sum will be 28; what is the number? 28 is $\frac{7}{5}$ of what number?

20. What is the number to which if its fifth be added the sum will be 54?

21. What is that number to which if its third and its fifth be added the sum will be 46?

22. What is the number to which if itself, its half and its tenth be added, the sum will be 52?

23. A girl being asked how many needles she had, answered, that if the number of the needles were increased by itself, one fourth of itself, and 4 needles, she should then have 85. How many had she?

24. What is the number to which if $\frac{2}{5}$ of $\frac{1}{2}$ of itself and $\frac{3}{10}$ of $\frac{1}{4}$ of itself be added the sum will be 100?

25. A farmer being asked how many sheep he had, answered, that if he had as many more, one half as many more, and 25 sheep, he should then have three times his present number; how many had he?

26. If a certain number be increased by itself, three fourths of itself, and five sixths of itself, the sum will be 14 more than three times the number; what is the number?

27. A boy being asked how old he was, said that if his age were increased by its third and its fourth and 5 years, the sum would be twice his age; what was his age?

28. What is that number from which if two fifths of itself be subtracted the remainder will be 18?

29. What is the number to which if five ninths of itself be added the sum will be 12 less than twice the number?

30. A farmer sells hay at 20 dollars a ton, which is $\frac{4}{5}$ of the cost; what is the cost, and how much does he gain on 6 tons?

31. A flour-dealer sold 75 barrels of flour for 450 dollars, which was $\frac{5}{6}$ of the cost; what did he gain on each barrel? What was the cost of 50 barrels of the same flour?

32. What is the number to which if we add seven eighths of itself, and from the sum subtract 15, the remainder will be 45?

33. A man sells nails for 5 cents a pound, which is $\frac{4}{5}$ of the cost; how many pounds are there in a cask of nails which yields him a profit of 75 cents?

34. What is the number from which if we subtract three sevenths of itself, and to the remainder add 12, the sum will be 40 ?

35. What is the number to which if we add itself, two sevenths of itself, and 14, the sum will be 6 less than three times the number ?

36. What is the number to which if we add itself, its half, its third, its fourth and its sixth, one third of the sum will be 78 ?

37. A gentleman spent three fifths of his ready money for a suit of clothes, one tenth for a hat, one twelfth for a pair of boots, and the remainder for books ; what part of his money did he spend for books ?

38. A lady paid one half of her ready money for a silk dress, one fifth of it for a bonnet, and one tenth of it for shoes ; with the remainder she bought three pairs of gloves, at 1 dollar a pair, and two bonnets for her daughters, at $3\frac{1}{2}$ dollars apiece ; how many dollars did she pay for the silk dress ?

39. When raisins are 12 cents a pound, five sixths of a pound of raisins will pay for five fourths of a pound of sugar ; how many pounds of sugar must be given for a cask of raisins containing 24 pounds ?

40. A trader bought 30 barrels of flour at 5 dollars a barrel, and sold it so as to gain one third of the cost. He received three fifths of the value in goods, and the rest in money ; how much money did he receive ?

41. What is the number to which if $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{5}{6}$ of itself be added, the sum will be $\frac{2}{3}$ of $\frac{9}{10}$ of 70 ?

42. A boy being asked how many questions there were in his arithmetic lesson, said that $\frac{3}{4}$ of the lesson exceeded $\frac{2}{3}$ of it by 4 questions ; how many questions were there ?

DENOMINATE NUMBERS.

SECTION XXXV.

DENOMINATE NUMBERS are those which express things of different kinds, or denominations; as 1 pound, 3 shillings, 5 pence.

English Money is the currency of England.

TABLE.

4 Farthings (qr.) . . . . = 1 Penny . . . . . d.
12 Pence . . . . = 1 Shilling . . . . s.
20 Shillings . . . . = 1 Pound . . . . . £.
21 Shillings . . . . = 1 Guinea.

1. How many farthings are there in 2s. 6d. ? SOLUTION. In 1 shilling there are 12 pence; in 2 shillings there are 2 times 12, or 24 pence; 24 pence plus 6 pence are 30 pence. In one penny there are 4 farthings; in 30 pence there are 30 times 4, or 120 farthings. ANS. 120 qr.

2. How many pounds are there in 89 shillings? SOLUTION. In 1 pound there are 20 shillings, and as many times as 20 shillings are contained in 89 shillings, so many pounds are there. 89 ÷ 20 = 4, 9 rem. ANS. £4 9s.

3. Change 102 pence to their value in shillings.

4. Change 75 qr. to their value in shillings, pence, and farthings.

5. In 5s. 8d. how many pence are there?

6. In ¾ of a £ how many times 10 pence are there?

7. What is the difference in farthings between ¾ of a £ and ¾ of a guinea?

8. What is the difference in pence between 1 shilling and 1 pound?

9. What is English money?

Federal Money is the currency of the United States.

TABLE.

10 Mills	= 1 Cent	. ct.
10 Cents	= 1 Dime	. d.
10 Dimes	= 1 Dollar	. $.
10 Dollars	= 1 Eagle	. E.

10. How many dollars are there in 275 cents?

11. How many cents are there in 5 dollars and 16 cents?

12. Change 415 cents to their value in dollars, dimes, and cents.

13. In 1 eagle how many cents are there? how many dimes? how many mills?

14. In 15 dimes how many times $\frac{1}{4}$ of a dollar are there?

15. If a gentleman exchanges 5 eagles for their equivalent in $2\frac{1}{2}$ dollar gold-pieces, how many does he receive?

16. What is Federal Money?

Troy Weight is used in weighing gold, silver, and precious stones.

TABLE.

24 Grains (gr.)	= 1 Pennyweight	. dwt.
20 Pennyweights	= 1 Ounce	oz.
12 Ounces	= 1 Pound	lb.

17. How many grains are there in 1 dwt.?

18. In 3 dwt. 6 gr. how many grains are there?

19. In 8 oz. 16 dwt. how many dwt. are there?

20. In 480 dwt. how many pounds are there?

21. $\frac{3}{4}$ of a pound plus 2 oz. are equal to how many dwt.?

22. $\frac{5}{8}$ of 3 dwt. are equal to how many grains?

23. $\frac{7}{8}$ of 2 oz. are how many dwt.? How many oz.?

24. When gold is worth $16\frac{3}{4}$ dollars an ounce, what is the value of a pound?

25. How is Troy Weight used?

Avoirdupois Weight is used for weighing all commodities except gold, silver, and precious stones.

TABLE.

16 Drams (dr.) . . . . . =	1 Ounce	oz.
16 Ounces . . . . =	1 Pound	lb.
28 Pounds . . . . =	1 Quarter	qr.
4 Quarters . . . . . . =	1 Hundred-weight	cwt.
20 Hundred-weight . . . =	1 Ton	T.

26. How many oz. are there in 16 dr. ? How many in 80 dr. ?

27. How many oz. are there in 68 dr. ?

28. In 3 oz. 9 dr. how many drams are there ?

29. In 4 qr. 15 lb. how many pounds are there ?

30. In 2 T. 12 cwt. 2 qrs. how many qrs. are there ?

31. 89 lbs. is what improper fraction of a quarter ? How many qrs. ?

32. 89 lbs. is what fraction of a hundred weight ?

33. What is the use of Avoirdupois Weight ?

34. By what weight would you buy nails ? hay ? silver ?

Apothecaries' Weight is used in mixing medicines, but not in selling them.

TABLE.

20 Grains (gr.) . . . . =	1 Scruple. . .	sc. or ℈.
8 Scruples . . . . . =	1 Dram . . .	dr. or ℨ.
8 Drams . . . . =	1 Ounce . . .	oz. or ℥.
12 Ounces . . . . =	1 Pound . . .	lb.

35. In 3 scruples how many grains are there ?

36. In 3 dr. how many scruples are there ?

37. 3 dr. 1 scruple 20 grains are how many times 22 grains ?

38. 86 gr. is what improper fraction of 1 ℈ ? how many ℈ ?

39. 99 ℨ is what fraction of a lb. ?

40. In $\frac{3}{4}$ of a lb. how many times 36 drams are there ?

41. In ¾ of an ounce how many times 10 grains?

42. What is the use of Apothecaries' Weight?

43. By what weight would you buy raisins? salts? pepper?

Cloth Measure is used in measuring cloths, &c.

TABLE.

2¼ Inches (in.) . . . . . = 1 Nail		na.
4 Nails . . . . , = 1 Quarter		qr.
4 Quarters . . . . . . = 1 Yard . . . . . ,		yd.
8 Quarters . . . . = 1 Flemish ell . . . Fl.		e.
5 Quarters . . . . = 1 English ell . . . E.		e.
6 Quarters . . . . = 1 French ell . . . Fr.		e.

44. In 3 yards how many quarters are there?

45. In 1 yd. how many nails are there?

46. At 5¼ dollars a yard, what will cost 2 qr. 3 na. of broadcloth?

47. In 32 nails how many qr. are there? how many Fl. e.? how many E. e.? how many Fr. e.?

48. How many inches are there in a yard of cloth?

49. What is the use of Cloth Measure?

Long Measure is used in measuring distances.

TABLE.

12 Inches (in.) . . . . . . = 1 Foot		ft.
3 Feet . . . . . . , . . . = 1 Yard . . .		yd.
6 Feet . . . . = 1 Fathom . .		fath.
5½ Yards, or 16½ Feet . . . = 1 Rod		rd.
40 Rods . . . . = 1 Furlong . . .		fur.
8 Furlongs . . . . = 1 Mile . . .		m.
3 Miles . . . . = 1 League . . .		lea.

OBS. The circumference of the earth is measured in degrees of latitude. 60 geographical miles, or 69¼ English miles, form 1 degree, and 360 degrees form the whole circumference.

50. In 8 ft. 7 in. how many inches are there?

51. In 2 fath. 1 ft. 9 in. how many inches are there?

52. How many feet of line will reach the bottom in 9¾ fathoms of water, the deck of the vessel being 15 feet above the water?

53. A school-room is 2 rods 15 feet in length, and 2¼ rods in breadth; how much does the length exceed the breadth?

54. What is the use of Long Measure?

55. Bunker Hill Monument is 220 feet high; what is its height in inches? what is its height in yards?

56. Joseph lives ¾ of a mile from the school-house; if he steps 10 times in travelling each rod, how many steps will he take in going to school?

Square Measure is used in measuring surfaces.

TABLE.

144	Square Inches (sq. in.) . . = 1 Square Foot . sq. ft.
9	Square Feet . . . . . . = 1 Square Yard sq. yd.
30¼	Sq. Yds., or 272¼ Sq. Ft. . = 1 Square Rod . sq. r.
40	Square Rods . . . . . . = 1 Rood . . . . R.
4	Roods . . . . = 1 Acre . . . . . A.
640	Acres . . . . = 1 Square Mile sq. m.

57. How many square inches are there in 2 sq. ft. 12 sq. in?

58. Change ¾ of a sq. ft. to sq. in., and add 6 sq. in. to it.

59. What will it cost to plaster a room containing 900 sq. ft., at 10 cents for every sq. yd.?

60. What is the cost of 240 sq. rods of land, at $1000 an acre?

61. The District of Columbia, containing the city of Washington, is 10 miles square; how many square acres does it contain?

62. What is the use of Square Measure?

63. By what measure would you ascertain the distance from your home to the school-room?

64. By what measure would you find the size of your slate?

Cubic Measure is used in measuring solidity or capacity.

TABLE.

1728	Cubic Inches (cub. in.) = 1 cubic foot . . cub. ft.
27	Cubic Feet . . . . . = 1 cubic yard. . cub. yd.
16	Cubic Feet . . . . . = 1 foot of wood.
8	Feet of Wood . . . . = 1 cord.

Obs. A *square* is 6 ft. long, 6 ft. wide, and 6 ft. deep, or 216 cub. ft. A *perch* is 1 ft. $\times$ 1½ ft. $\times$ 16½ ft., or 24¾ cub. ft.

65. 67 cubic feet are equal to how many feet of wood?

66. How many cubic feet are equal to 1 cord of wood?

67. What is the price of 2 ft. of wood, at the rate of 9 dollars a cord?

68. What is the use of Cubic Measure?

Liquid Measure is used in measuring liquids.

TABLE.

4	Gills (gi.) . . . . . . = 1 Pint . . . . . pt.
2	Pints . . . . = 1 Quart . . . . qt.
4	Quarts . . . . = 1 Gallon . . . . gal.
31½	Gallons . . . . = 1 Barrel . . . . bl.
63	Gallons . . . . = 1 Hogshead . . . hhd.

Obs. The English wine gallon contains 231 cubic inches. In some places milk and malt liquors are sold by a measure which contains 282 cubic inches in a gallon. A pint of water is estimated to weigh a pound.

69. How many quarts are there in a barrel?

70. How many barrels are there in a hogshead?

71. How many gills are there in $\frac{3}{16}$ of 2 gallons?

72. Change 129 quarts to their value in barrels, gallons, and quarts.

73. How many pounds of water can a boy carry in a pail holding 2½ gallons?

74. What is the use of Liquid Measure?

Dry Measure is used in measuring coal, lime, fruit, and other goods commonly sold by heap measure.

TABLE.

2 Pints (pt)	. . . . . = 1 Quart	. . . . .	qt.
8 Quarts	. . . . = 1 Peck	. . . . . .	pk.
4 Pecks	. . . . . . = 1 Bushel	. . . . .	bu.
36 Bushels	. . . . . . = 1 Chaldron	. . . .	ch.

75. In 48 pints how many pecks are there?

76. In 3 chaldrons of coal how many bushels are there?

77. What is the use of Dry Measure?

78. By what measure would you buy potatoes? corn? oil? boards? ribbon?

— TIME.

60 Seconds (sec.)	. . . . . = 1 Minute	. . .	min.
60 Minutes	. . . . = 1 Hour	. . . .	hr.
24 Hours	. . . . = 1 Day	. . . .	day
7 Days	. . . . = 1 Week	. . . .	wk.
365 Days	. . . . = 1 Year	. . . .	Y.

Obs. The solar year contains 365 days, 5 hours, 48 minutes, and 48 seconds, or nearly ¼ of a day more than 365 days. Every fourth year is therefore called *leap year*, and contains 366 days. The leap years are those which are exactly divisible by 4. A common month contains 4 weeks.

79. How many seconds are there in 3 min. 48 sec.?

80. In ⅔ of a day how many hours are there?

81. In 37 days there are how many weeks? months?

82. What is the length of the solar year? leap year?

83. How can you tell which are the leap years?

CIRCULAR MEASURE.

60 Seconds (″)	. . . . . = 1 Minute	. . . . .	″
60 Minutes	. . . . = 1 Degree	. . . . .	°
30 Degrees	. . . . . . = 1 Sign	. . . . .	s.
12 Signs, or 360°	. . . . = 1 Circle	. . . .	cir.

84. How many degrees are there in ½ the circumference of the earth? in ¼ of the circumference?

DIFFERENCE IN TIME BETWEEN DIFFERENT PLACES.

SECTION XXXVI.

THE earth is 360 degrees in circumference, and turns on its axis every 24 hours; it consequently passes through $\frac{1}{24}$ of 360 degrees, or 15 degrees, in an hour, which is 1 degree in 4 minutes.

OBS. When it is noon at any particular place, it is after noon east of that place, and before noon west of that place.

1. The longitude of Boston is 71° W., and that of Chicago 87½° W.; what is the difference of time between the two places? SOLUTION. · 87½° — 71° = 16½°. The difference in time is 4 minutes to each degree. 16½ times 4 min. = 66 min., or 1 hour 6 min. ANS. 1 hour 6 min.

2. What o'clock is it in Boston when it is noon in Chicago?

3. What time is it in Chicago when it is noon in Boston?

4. A gentleman travels west from Boston to Louisville, in Kentucky, and his watch keeps accurately Boston time; is it too fast or too slow by the Louisville time? how much so, the difference in longitude being 14½ degrees?

5. What time is it in Louisville when it is noon in Boston? what time is it in Boston when it is noon in Louisville?

6. Portsmouth, N. H., is in the same longitude with Boston, and St. Louis is in 90½ degrees west longitude; what is the difference of time between Portsmouth and St. Louis?

7. What time is it in Boston when it is 7 o'clock, A. M., in St. Louis? what time is it in St. Louis when it is noon in Boston?

8. The difference of time between two places is 5 h. 20 min.; what is the difference of longitude? (5⅓ × 15° = 80°.)

9. If the difference of longitude between two places is 80°, what is the difference of time?

10. What is the difference of longitude, where the difference of time is 2 hours 20 min.? 6 hours 48 min.? 7 hours 24 min.?

11. The capital of the United States is 77 degrees west of London; what is the difference of time between the two cities?

12. If the difference of time between Madrid and Washington is 4 hours 56 min., what is the longitude of Madrid?

13. If the difference of time between St. Petersburg and Boston is 7 hours 8 min., what is the longitude of St. Petersburg?

14. What is the time in Washington when it is noon in St. Petersburg?

15. Eastport, Maine, the most eastern seaport of the United States, is in 67° W. longitude, and San Francisco, California, the most western, is in 124° W.; what is the difference in time?

16. What time is it in San Francisco when it is noon in Eastport?

17. What time is it in Eastport when it is noon in San Francisco?

18. The longitude of Liverpool is 3° W., and that of New York 74° W.; what is the difference of time between the two cities?

19. New Orleans is in 90° W. longitude; what time is it in New Orleans when it is noon in Liverpool?

20. What time is it in Liverpool when it is noon in New Orleans?

21. How many degrees of longitude make 1 hour's difference in time? why?

DIFFERENCE OF TIME BETWEEN TWO DATES.

SECTION XXXVII.

SUBTRACT the first date from the last, taking days from days, months from months, and years from years.

OBS. January is the 1st month, February the 2nd, March the 3d, &c.

OBS. In finding the difference between two dates, each month is usually reckoned as 30 days.

. 1. What is the difference of time between January 4 and September 18, of the same year?

SOLUTION. 4 days from 18 days leave 14 days; January is the 1st month, September the 9th month; $9 - 1 = 8$. ANS. 8 months, 14 days.

2. What is the difference of time between February 18 and November 10, of the same year?

SOLUTION. As I cannot subtract 18 days from 10 days, I borrow 30 days, or 1 month, from the months in the minuend, and subtract 18 days from that; $30 - 18 = 12$; $12 + 10$ (the number of days in the minuend) $= 22$. February is the 2nd month, and November is the 11th month: $11 - 1$ (the month that was borrowed) $= 10$; $10 - 2 = 8$. ANS. 8 months, 22 days.

3. What is the difference of time between July 4, 1845, and September 6, 1848?

4. What is the difference of time between August 9, 1841, and October 1, 1848?

5. What is the difference of time between November 29, 1842, and April 4, 1846?

6. What is the difference of time between May 18, 1842, and December 2, 1847?

7. What is the difference of time between June 6, 1850, and February 4, 1852?

8. What is the difference of time between January 8, 1852, and January 4, 1853?

9. What is the difference of time between August 1, 1840, and May 1, 1853?

10. What is the difference of time between April 12, 1848, and June 6, 1852?

11. What is the difference of time between May 10, 1843, and April 8, 1853?

12. What is the difference of time between June 6, 1849, and May 3, 1853?

13. What is the difference of time between January 12, 1835, and October 8, 1852?

14. What is the difference of time between September 18, 1846, and February 24, 1853?

15. The pilgrims landed at Plymouth, in Massachusetts, December 22, 1620; the first settlement at Boston, the capital of the state, was made September 17, 1630; required the difference of time between the settlement of the two places.

16. America was declared independent July 4, 1776; the treaty of peace with Great Britain was signed Sept. 23, 1783; how long a time elapsed between the declaration and peace?

17. The last war with Great Britain was declared June 12, 1812; the treaty of peace was signed December 24, 1814; what was the length of the war?

18. Washington was inaugurated President of the United States April 30, 1789; President Pierce was inaugurated March 4, 1853; what is the difference of time between the inauguration of the two Presidents?

19. The first book was printed August 14, 1457; required the time that has elapsed since.

20. A gentleman gives his note for value received, March 18, 1858, and promises to pay February 10, 1862; in what time will the note be payable?

21. How do you find the difference of time between two dates?

DECIMALS.

SECTION XXXVIII.

A PECIMAL FRACTION is one or more equal parts of a unit divided into tenths, hundredths, &c. It is usual to write only the numerator of a decimal fraction, with a decimal point prefixed. The denominator is 1, with as many ciphers annexed as there are figures in the numerator.

The first place at the right of the point 'is the tenths' place, the second is the hundredths' place, the third is the thousandths' place, &c.

We read a decimal as if it were a whole number, and finish by giving the denominator.

This is enumerated thus: 6 tenths, 7 hundredths, 9 thousandths, 4 ten-thousandths, 5 hundred-thousandths, 3 millionths, 2 ten-millionths. It is read thus: six millions, seven hundred ninety-four thousand, five hundred thirty-two *ten-millionths*. The lowest denomination expressed is the denomination of the whole.

Tenths. Hundredths. Thousandths. Ten-thousandths. Hundred-thousandths. Millionths. Ten-millionths.
.6 7 9 4 5 3 2

In decimals, as in whole numbers, any figure moved one place to the left expresses ten times its former value. .05 × 10 = .50 = .5.

1. In what place from the decimal point must 2 be written to express 2 tenths? ANS. First place. To express 2 hundredths? to express 2 thousandths?

2. In what place from the decimal point must 7 be written to express 7 hundredths? to express 7 ten-thousandths? to express 7 millionths?

3. What part of unit would a figure express in·

the second place to the right of the decimal point? in the third place? in the fifth place?

4. In which place to the *left*, in whole numbers, do we express hundreds? in which place to the *right* of the point in decimals do we express hundredths?

5. In which place to the left in whole numbers do we express thousands? in which place to the right of the point in decimals do we express thousandths?

6. In which place to the left in whole numbers must 6 be written to express six tens of thousands, and what places must be filled with ciphers?

7. In which place to the right of the point in decimals must 6 be written to express 6 ten thousandths, and what places must be filled with ciphers?

8. Enumerate and read the following decimals:

.4	.67
.45	.504
.045	.9625
.0456	.87
.00456	.942
.000456	.67315

Any common fraction may be reduced to an equivalent decimal by annexing ciphers to the numerators, and dividing by the denominator.

Reduce $\frac{1}{2}$ to a decimal form. SOLUTION. Annex a cipher to the numerator 1, and we have 10; 10 ÷ 2 = 5. ANS. .5. Again, in 1 there are $\frac{10}{10}$, and $\frac{1}{2}$ of $\frac{10}{10}$ = $\frac{5}{10}$. ANS. .5.

9. Reduce $\frac{1}{4}$ to a decimal expression.

10. Reduce $\frac{1}{5}$ to a decimal expression.

11. Reduce $\frac{1}{10}$ to a decimal expression.

12. Reduce $\frac{1}{20}$ to a decimal expression.

13. Reduce $\frac{1}{25}$ to a decimal expression.

14. Reduce $\frac{1}{50}$ to a decimal expression.

15. Reduce $\frac{3}{4}$ to a decimal expression.

16. Reduce $\frac{2}{5}$ to a decimal expression.

17. Reduce ⅛ to a decimal expression.

A decimal fraction may be brought to a common fraction by expressing the decimal denominator, and then reducing it to its lowest terms.

18. Reduce .4 to a common fraction. (₄₁₀ = ⅖.)

19. Reduce .75 to a common fraction.

20. Reduce .8 to a common fraction.

21. Change to fractions the following decimals: .6, .18, .25, .75, .125, .175, .225, and .88.

In addition or subtraction of decimals, we add or subtract tenths, hundredths, &c., separately, in the same manner that we add or subtract whole numbers.

22. What is the sum of .1 and .05 ?

23. What is the sum of .6 and .25 ?

24. What is the sum of .6 and .35 ?

25. What is the sum of .6 and .45 ?

26. What is the difference between .1 and .05 ?

27. What is the difference between .6 and .25 ?

28. What is the difference between .6 and .35 ?

29. What is the difference between .6 and .45 ?

30. What is the sum of .06 and .005 ?

31. What is the sum of .06 and .015 ?

32. What is the sum of .06 and .025 ?

33. What is the sum of .06 and .035 ?

34. What is the sum of .06 and .045 ?

35. What is the sum of .06 and .055 ?

In multiplication of decimals we proceed as in multiplication of whole numbers, and from the right of the product point off for a decimal as many figures as there are decimal places in both multiplier and multiplicand taken together.

36. Multiply 7 by .16. SOLUTION. 16 times 7 are 112; there being two decimal places in both factors, we must point off two from the product. ANS. 1.12.

37. Multiply 16 by .7; 19 by .5; 20 by .8; 15 by .9.

38. Multiply, 8 by .14; 16 by .12; 18 by .07; 16 by .09.

39. Multiply .15 by .9; .25 by .8; .95 by .5; .4 by .44.

40. Multiply .12 by .12; .15 by .05; 18 by .04; 17 by .03.

41. Multiply 2.1 by 5; 2.1 by .5; 2.1 by .0; 2.15 by 1.2.

42. Multiply 3.2 by 4; 4.1 by 6; 3.2 by .4; 2.2 by 8.

43. Multiply 8 by 1.2; 12 by 1.1; 14 by .9; 1.8 by 6; .7 by .7.

44. Multiply 1.25 by .8; 1.75 by .6; 11.4 by .3; 12.1 by .2.

45. Multiply .1 by .01; .01 by .01; .6 by .06; .09 by .09.

46. What is a decimal fraction?

47. How is it written?

Obs. The decimal point is always understood to be written at the right of a whole number, and the value of each place in whole numbers, as well as in decimals, is determined by its distance from the decimal point.

48. What is the denominator of a decimal fraction?

49. What is the first place at the right of the decimal point? the second place, &c.?

50. Enumerate the decimal table as far as ten-millionths.

51. How do we read a decimal?

52. What is the denomination of whole decimals?

53. How does it affect the value of a decimal figure to move it one place to the left? one place to the right?

54. How may any common fraction be brought to a decimal?

55. How may a decimal be brought to a common fraction?

56. How do we add or subtract decimals? how do we multiply by a decimal?

PERCENTAGE.

SECTION XXXIX.

PERCENTAGE, or *per cent.*, denotes any number of *hundredths.* 1 per cent. is written .01; 2 per cent. is written .02; 5 per cent. is written .05, &c.

In mental arithmetic, when there is a fraction of 1 per cent., it is more convenient to find the per cent. first, and then add the fractional part of 1 per cent. of the number to the product. . Thus, to find $7\frac{1}{2}$ per cent. of 200 : 7 per cent. of 200 = 14, 1 per cent. of 200 = 2, and therefore $\frac{1}{2}$ per cent. of 200 = 1 : 14 + 1 = 15. ANS. 15.

1. What is 6 per cent. of 20? SOLUTION. 20 × .06 = 1.2. ANS. 1.2.

2. What is 6 per cent. of 30? SOLUTION. $\frac{1}{100}$ of 30 = $\frac{30}{100}$ or $\frac{3}{10}$; $\frac{3}{10}$ × 6 = $\frac{18}{10}$ = $1\frac{8}{10}$. ANS. 1.8.

OBS. The pupil should be required to give the explanation in this manner, occasionally, to show that he understands the principle.

3. What is 2 per cent. of 12? 28? 45? 90?

4. What is 6 per cent. of 40? 45? 60? 75? 90?

5. What is 5 per cent. of 30? 50? 75? 90? 120?

6. What is 4 per cent. of 45? 65? 70? 150? 200?

7. What is 3 per cent. of 60? 80? 100? 140? 160?

8. What is 7 per cent. of 12? 25? 30? 60? 180?

9. What is 8 per cent. of 15? 35? 65? 80? 90?

10. What is 10 per cent. of 60? 150? 180? 190? 200?

11 What is 11 per cent. of 60? 70? 80? 90? 100?

12. What is $3\frac{1}{2}$ per cent. of 60? SOLUTION. 3 per cent. of 60 = 1.8, $\frac{1}{2}$ per cent. of 60 = .3; 1.8 + .3 = 2.1. ANS. 2.1.

13. What is 3½ per cent. of 80 ? 40 ? 20 ? 100 ?

14. What is 5½ per cent. of 60 ? 80 ? 40 ? 100 ? 120 ?

15. What is 7½ per cent. of 60 ? 80 ? 100 ? 120 ? 140 ?

16. What is ¼ of 1 per cent. of 60 ? SOLUTION. 1 per cent. of 60 = .6 : ¼ of 1 per cent. of 60 = .6 × ¼ = .15 ($\frac{6}{10}$ × ¼ = $\frac{6}{40}$ = $\frac{3}{20}$ = .15). Ans. 15.

17. What is ¼ of 1 per cent. of 80 ? 100 ? 120 ? 160 ? 200 ?

OBS. This sign, $, is read dollars.

18. A man subscribed for 15 shares of the stock in a proposed railroad, at $100 a share ; how much would an assessment of 5 per cent. on his stock amount to ?

19. A man's income is $1200, and he saves 9 per cent. of it ; how much does he save in a year ?

20. What per cent. of 50 is 6 ? SOLUTION. 1 is $\frac{1}{50}$ of 50, and therefore 6 is $\frac{6}{50}$ or $\frac{3}{25}$ of 50 ; $\frac{3}{25}$ reduced to a decimal expression (to per cent.) = .12. Ans. 12 per cent.

OBS. Two ciphers are annexed to the numerator of the fraction when it is brought to per cent.

21. 4 is what per cent. of 50 ?

22. 4 is what per cent. of 60 ? SOLUTION. 4 is $\frac{4}{60}$ or $\frac{1}{15}$ of 60 ; $\frac{1}{15}$ reduced to per cent. (100 ÷ 15 = $6\frac{10}{15}$ or $\frac{2}{3}$) = $6\frac{2}{3}$. Ans. $6\frac{2}{3}$ per cent.

23. 4 is what per cent. of 80 ?

24. 15 is what per cent. of 60 ?

25. 5 is what per cent. of 20 ?

26. 10 is what per cent. of 20 ?

27. 6 is what per cent. of 60 ?

28. 8 is what per cent. of 80 ?

29. 16 is what per cent. of 80 ?

30. 40 is what per cent. of 80 ?

31. 60 is what per cent. of 80 ?

Obs. The following table is given for the convenience of the pupil.

$$5 \text{ per cent. or } \tfrac{5}{100} = \tfrac{1}{20}.$$
$$6 \text{ per cent. or } \tfrac{6}{100} = \tfrac{3}{50}.$$
$$10 \text{ per cent. or } \tfrac{10}{100} = \tfrac{1}{10}.$$
$$12 \text{ per cent. or } \tfrac{12}{100} = \tfrac{3}{25}.$$
$$15 \text{ per cent. or } \tfrac{15}{100} = \tfrac{3}{20}.$$
$$20 \text{ per cent. or } \tfrac{20}{100} = \tfrac{1}{5}.$$
$$25 \text{ per cent. or } \tfrac{25}{100} = \tfrac{1}{4}.$$
$$30 \text{ per cent. or } \tfrac{30}{100} = \tfrac{3}{10}.$$
$$33\tfrac{1}{3} \text{ per cent. or } \tfrac{33\frac{1}{3}}{100} = \tfrac{1}{3}.$$
$$37\tfrac{1}{2} \text{ per cent. or } \tfrac{37\frac{1}{2}}{100} = \tfrac{3}{8}.$$
$$40 \text{ per cent. or } \tfrac{40}{100} = \tfrac{2}{5}.$$
$$50 \text{ per cent. or } \tfrac{50}{100} = \tfrac{1}{2}.$$
$$60 \text{ per cent. or } \tfrac{60}{100} = \tfrac{3}{5}.$$
$$75 \text{ per cent. or } \tfrac{75}{100} = \tfrac{3}{4}.$$

82. At what part of their cost must goods be sold in order to gain 20 per cent. ? SOLUTION. 20 per cent. = $\tfrac{1}{5}$; therefore the price of the goods must be increased $\tfrac{1}{5}$, which will make it $\tfrac{6}{5}$ of their cost. ANS. $\tfrac{6}{5}$.

33. At what part of their cost must goods be sold in order to gain 10 per cent. ? 25 per cent. ? 30 per cent. ? 33$\tfrac{1}{3}$ per cent. ? 37$\tfrac{1}{2}$ per cent. ? 40 per cent. ?

34. If a yard of cloth cost 4 dollars, at what price must it be sold to gain 10 per cent. ? 25 per cent. ? 30 per cent. ?

35. At what part of their cost must goods be sold in order to gain 33$\tfrac{1}{3}$ per cent. ? 37$\tfrac{1}{2}$ per cent. ?

36. If a trader buy sugar at 5 cents per lb., at what price per lb. must he sell it in order to gain 10 per cent. ? ($\tfrac{11}{10}$ of $\tfrac{5}{1}$.) 20 per cent. ? 25 per cent. ?

37. What are $1\tfrac{1}{10}$ of $\tfrac{5}{1}$? $\tfrac{6}{5}$ of 5? $\tfrac{5}{4}$ of 5?

38. If a barrel of flour cost 4 dollars, at what price must it be sold in order to gain 6 per cent. ? SOLUTION. 6 per cent. of 400 is 24, 400 + 24 = 424. ANS. $4.24.

39. By what part of the cost must the price of goods be increased to gain 20 per cent. ?

40. By what part of the cost must the price of goods be diminished to lose 20 per cent. ? for what part of their cost must they be sold ?

41. When the trade price of a book is 60 cents, for what part of the price is it sold, when the discount is 20 per cent. ? at what price ?

42. If the price of a book is 75 cents, what is received for it when it is sold at a discount of 37½ per cent. ? when the discount is 33⅓ per cent. ?

43. 3 is what per cent. of 5 ? of 6 ? of 8 ? of 9 ?

44. If a trader should buy sugar at 5 cents per pound, and sell it at 8 cents, what would he gain per cent. ? SOLUTION. If he pays 5 cents per lb., and sells it for 8 cents per lb., he gains 3 cents or $\frac{3}{5}$ of the cost; $\frac{3}{5} = \frac{60}{100}$. ANS. 60 per cent.

45. If a trader should buy sugar at 5 cents a pound, at what price must he sell it per lb., in order to gain 60 per cent. ?

46. If a trader should buy sugar at 6 cents per lb., and sell it at 9 cents, what would he gain per cent. ?

47. If a trader should buy sugar at 7 cents per lb., at what price must he sell it to gain 40 per cent. ? to gain 33⅓ per cent. ? 37½ per cent. ?

48. When Manilla coffee is purchased for 10 cents per lb. and sold for 8 cents per lb., what is the loss per cent. ?

49. If Manilla coffee is bought for 10 cents per lb. and sold for 15 cents per lb., what is the gain per cent. ?

50. When mackerel are bought for $12 a barrel and sold at an advance of 6 per cent., what is the selling price ? when the advance is 10 per cent. ? 12 per cent. ? 20 per cent. ? 33⅓ per cent. ? 37½ per cent. ?

51. A trader buys sugar at 6¼ cents per lb. ; at what price must he sell it per lb. to gain 20 per cent. ?

52. What is meant by percentage ?

INTEREST.

SECTION XL.

INTEREST is a certain per cent. of a sum of money paid for its use.

The sum on which the interest is computed is called the *principal*. The principal and interest, added together, is called the *amount*.

The per cent. is called the *rate*, and is always reckoned by the year.

At 6 per cent., the interest on any sum of money for one year is $\frac{6}{100}$ (.06) or $\frac{3}{50}$ of the principal; at 5 per cent., the interest is $\frac{5}{100}$ (.05) or $\frac{1}{20}$ of the principal; at 4 per cent., the interest is $\frac{4}{100}$ (.04) or $\frac{1}{25}$ of the principal.

The interest on any sum of money, at 6 per cent., for one month, is $\frac{1}{12}$ of $\frac{6}{100}$ or $\frac{1}{200}$ of the principal; for 1 day, it is $\frac{1}{30}$ of $\frac{1}{200}$ or $\frac{1}{6000}$ of the principal.

OBS. In computing interest, 30 days are reckoned a month, and 12 months a year.

If the interest, at 6 per cent., for 1 month, is $\frac{1}{200}$ of the principal, for 100 months, it is $\frac{100}{200}$, or $\frac{1}{2}$ of the principal; for 50 months, it is $\frac{50}{200}$ or $\frac{1}{4}$ of the principal; for 20 months, it is $\frac{20}{200}$ or $\frac{1}{10}$ of the principal; for 10 months, it is $\frac{10}{200}$ or $\frac{1}{20}$ of the principal.

1. What part of the principal is the interest for 30 months? for 40 months? for 60 months?

2. What part of the principal is the interest for 5 months? for 15 months? for 25 months? for 35 months?

3. What part of the principal is the interest for $2\frac{1}{2}$ months? for $7\frac{1}{2}$ months? for $12\frac{1}{2}$ months? for $15\frac{1}{2}$ months?

4. What part of the principal is the interest for 2

months? for 3 months? for 4 months? for 6 months? for 7 months? for 8 months? for 9 months? for 11 months? for 13 months? for 14 months? for 17 months?

In seeking what part of the principal equals the interest for any given rate and time, the pupil should be taught to form such fractional numbers as can be most conveniently used as multipliers. It is sometimes more convenient to use a common fraction, and sometimes a decimal. Thus, for 50 months, it is easier to take one quarter of the principal than to multiply by .25.

The interest on any sum of money, for 4 years and 2 months, is $\frac{1}{4}$ of the principal; for 3 years and 4 months, it is $\frac{1}{5}$ of the principal; for 1 year and 8 months, it is $\frac{1}{10}$ of the principal; for 2 years and 1 month, it is $\frac{1}{8}$ of the principal; for 3 years and 9 months, it is $\frac{1}{5}$ and $\frac{1}{10}$ of the principal; for 2 years and 8 months, it is $\frac{1}{10}$ and $\frac{6}{100}$ of the principal; for 2 years and 10 months, it is $\frac{1}{10}$ and $\frac{7}{100}$ of the principal.

OBS. When the per cent. is not specified, it is always considered to be 6 per cent.

5. What part of the principal is the interest for 30 days? for 20 days? for 10 days? for 5 days?

6. What part of the principal is the interest for 15 days? for 25 days? for 35 days? for 45 days?

7. What part of the principal is the interest for 1 day? for 2 days? for 3 days? for 4 days? for 6 days? for 7 days? for 9 days? for 11 days?

8. What is the interest of 20 dollars, for 2 years and 6 months? SOLUTION. 2 years and 6 months are 30 months. The interest for 30 months is $\frac{30}{200}$, or $\frac{3}{20}$ of the principal; $\frac{1}{20}$ of 20 dollars is 1 dollar, $\frac{3}{20}$ are 3 times 1 dollar, or 3 dollars. ANS. 3 dollars.

9. What is the interest of 50 dollars, for 4 years and 2 months? for 3 years and 4 months? for 1 year and 8 months?

10. What is the interest of 60 dollars, for 5 years? for 6 years and 3 months? for 3 years and 9 months?

11. What is the interest of 70 dollars, for 1 year and 3 months? for 2 years and 1 month?

12. What is the interest of 100 dollars, for 16 months? for 18 months? for 20 months? for 28 months?

13. What is the interest of $70, for 2 years and 8 months?

14. What is the interest of $75, for 4 years and 6 months?

15. What is the interest of $150, for 6 years and 3 months?

16. What is the interest of $300, for 5 years and 5 months?

17. What is the interest of $150, for 2 years and 8 months?

18. What is the interest of $90, for 3 years and 10 months?

19. What is the interest of $110, for 3 years and 4 months? what is the amount?

20. What is the interest of $120, for 4 years and 6 months?

21. What is the interest of $125, for 4 years and 10 months?

22. What is 1 per cent. of $150? what is $\frac{1}{2}$ of 1 per cent. of $150?

23. What is the interest of $150, for 4 years and 5 months?

24. What is the interest of $200, for 5 years and 5 months?

25. What is the interest of $160, for 5 years and 5 months?

26. What is the interest of $180, for 5 years and 7 months?

27. What is the interest of $190, for 6 years and 8 months?

28. What is the interest of $1000, for 1 year? for 2 years and 3 months? for 3 years and 4 months? for 5 years and 11 months?

29. What is the interest of $1, for 12 days? for 18 days? for 15 days?

30. What is the interest of $80, for 12 days? for 18 days? for 21 days?

31. What is the interest of $75, for 2 months and 2 days? for 6 months and 15 days?

32. What is the interest of $90, for 1 year, 4 months, and 10 days?

33. What is the interest of $80, for 2 years, 6 months, and 9 days, at 6 per cent.? what is the amount?

34. What is the interest of $70, for 4 years and 8 months? what is the amount?

The interest of 1 dollar, at 6 per cent., for 1 year, is $\frac{6}{100}$ of 1 dollar, or 6 cents. The interest of 1 dollar, for 1 month, is $\frac{1}{12}$ of 6 cents, which is $\frac{6}{12}$ or $\frac{1}{2}$ of a cent. The interest of 1 dollar, for 1 day, is $\frac{1}{30}$ of $\frac{1}{2}$ of a cent, which is $\frac{1}{60}$ of a cent, or $\frac{1}{6000}$ of a dollar, or $\frac{1}{6}$ of a mill. The interest, therefore, of any number of dollars, at 6 per cent., is 6 times as many cents as there are years, one half as many cents as there are months, and $\frac{1}{6}$ as many mills as there are days.

Obs. Those who prefer can find the interest on 1 dollar for the time, and multiply this by the principal.

35. What is the interest of 80 dollars, for 2 years, 4 months, and 12 days, at 6 per cent.? Solution. 2 years and 4 months are 28 months. . The interest of 1 dollar, for 28 months, is 14 cents ($\frac{1}{2}$ as many cents as there are months). The interest of 1 dollar, for 12 days, is 2 mills ($\frac{1}{6}$ as many mills as there are days). The interest of 1 dollar, for 2 years, 4 months, and 12 days, is 14

cents and 2 mills (.142). The interest of 80 dollars, for the same time, is 80 times .142 = $11.360. · Ans. $11.36.

Obs. The following questions may be performed by either method.

36. What is the interest of ·$50, for 1 year, 6 months, and 15 days?

37. What is the interest of $75, for 2 years, 9 months, and 18 days?

38. What is the interest of $90, for 3 years, 8 months, and 21 days?

39. What is the interest of $120, for 4 years, 10 months, and 24 days?

Obs. When the rate is other than 6 per cent., first find the interest at 6 per cent., and then take as many sixths of that as the per cent. indicates. Thus, 5 per cent. is $\frac{5}{6}$ of 6 per cent.

40. What is the interest of 200 dollars, for 2 years and 9 months, at 5 per cent.? what is the amount?

41. What is the interest of 300 dollars, for 3 years and 8 months, at 4 per cent.? what is the amount?

42. What is the interest of 400 dollars, for 4 years and 10 months, at 3 per cent.? what is the amount?

43. What is the interest of 500 dollars, for 2 years, 6 months, and 20 days, at 8 per cent.? what is the amount?

44. What is the interest of 600 dollars, for 1 year, 4 months, and 20 days, at 6 per cent.? what is the amount?

45. What is the interest of 600 dollars, for 1 year, 4 months, and 20 days, at 5 per cent.?

46. What is interest?

47. What is the principal? what is the amount?

48. What is the rate?

49. In computing interest, how many days are reckoned a month? how many months a year?

SECTION XLI.

1. WHAT principal, at 6 per cent., will gain $9 in 1 year and 6 months? SOLUTION. The interest of $1, for 1 year and 6 months, is 9 cents; it will require as many dollars to gain $9 in 1 year and six months as 9 cents are contained times in 900 cents: 9 cents are contained in 900 cents 100 times. ANS. $100.

2. What principal, at 6 per cent., will gain $10 in 2 years and 6 months?

3. What principal, at 6 per cent., will gain 1 dollar in 1 year and 4 months?

4. What principal, at 6 per cent., will gain $56 in 4 years and 8 months?

5. What principal, at 6 per cent., will gain $206 in 5 years and 8 months?

6. What principal, at 6 per cent., will gain $80 in 8 years and 4 months?

7. In what time will $100 gain $100, at 6 per cent.? SOLUTION. Any sum of money, at 6 per cent., will gain .06 or $\frac{3}{50}$ of itself in 1 year; to gain a sum equal in value to itself, it would require as many years as $\frac{3}{50}$ are contained times in $\frac{50}{50}$; $\frac{3}{50}$ in $\frac{50}{50}$, $16\frac{2}{3}$ times. ANS. $16\frac{2}{3}$ years.

8. In what time will $1000 gain $1000, at 4 per cent.?

9. In what time will $500 gain $250, at 5 per cent.?

10. In what time will $700 gain $175, at 10 per cent.?

11. In what time will $700 gain $175, at 8 per cent.?

12. In what time will $1000 gain $125, at $12\frac{1}{2}$ per cent.?

13. When the rate per cent. is known, how can you

8

find the time in which any sum of money will gain a sum equal to itself?

14. At what rate per cent. will $10 gain $2 in 5 years? SOLUTION. The gain is $\frac{2}{10}$ or $\frac{1}{5}$ of the principal; if the gain is $\frac{1}{5}$ of the principal in 5 years, in 1 year it will be $\frac{1}{5}$ of $\frac{1}{5}$, or $\frac{1}{25}$ of the principal; $\frac{1}{25}$ = .04. ANS. 4 per cent.

15. At what rate per cent. will $50 gain $8 in 3 years? The gain is $\frac{8}{50}$ or $\frac{4}{25}$ of the principal; if the gain is $\frac{4}{25}$ of the principal in 3 years, in 1 year it will be $\frac{1}{3}$ of $\frac{4}{25}$, or $\frac{4}{75}$ of the principal; $\frac{4}{75}$ = .05$\frac{1}{3}$. ANS. 5$\frac{1}{3}$ per cent.

16. At what rate per cent. will $75 gain $25 in 8 years?

17. At what rate per cent. will $80 gain $16 in 4 years?

18. At what rate per cent. will $125 gain $25 in 10 years?

19. At what rate per cent. will $100,000 gain $1000 in 1 year?

20. At what rate per cent. will $72 gain $27 in 6 years?

21. What principal, at 6 per cent., will amount to $560 in 2 years? SOLUTION. The interest, for 2 years, is $\frac{12}{100}$ or $\frac{3}{25}$ of the principal; the amount, therefore, is $\frac{25}{25} + \frac{3}{25} = \frac{28}{25}$ of the principal. If $560 is $\frac{28}{25}$, $\frac{1}{25} = \frac{1}{28}$ of $560, and $\frac{25}{25} = \frac{25}{28}$ of $560 = $560. ANS. $500.

22. What principal, at 6 per cent., will amount to $460 in 2 years and 6 months?

23. What principal, at 6 per cent., will amount to $2800 in 6 years and 8 months?

24. What principal, at 6 per cent., will amount to $2500 in 4 years and 2 months?

25. What principal, at 6 per cent., will amount to $150 in 8 years and 4 months?

RATIO.

SECTION XLII.

RATIO is the relation which one quantity bears to another, of the same kind, with respect to magnitude.

The ratio of two numbers is the quotient of the first divided by the second. Thus, the ratio of 12 to 4 is 3, since $12 \div 4 = \frac{12}{4} = 3$; and the ratio of 4 to 12 is $\frac{1}{3}$, since $4 \div 12 = \frac{4}{12} = \frac{1}{3}$.

If both terms of the ratio be multiplied or divided by the same number, the ratio will not be changed; thus, 12 : 4 (the ratio of 12 to 4) is the same as 6 : 2, or 24 : 8. This is evident from the fact that the terms of a ratio are the terms of a fraction, which may be multiplied or divided by the same number, without changing the value of the fraction.

Proportion is the union of two equal ratios.

Fractions having the same denominator are to each other as their numerators. Thus, the ratio of $\frac{2}{5}$ to $\frac{3}{5}$ is the same as the ratio of 2 to 3.

1. What is the ratio of 4 to 5 ? ANS. $\frac{4}{5}$.

2. What is the ratio of 7 to 8 ? 7 is what part of 8 ?

3. What is the ratio of 9 to 12 ? 9 is what part of 12 ?

4. What is the ratio of 8 to 4 ? 8 is what part of 4 ?

5. What is the ratio of $\frac{3}{5}$ to $\frac{4}{5}$? 3 is what part of 5 ?

6. What is the ratio of $\frac{50}{100}$ to $\frac{25}{100}$? 50 is what part of 25 ?

7. What is the ratio of $\frac{3}{4}$ to $\frac{5}{6}$? SOLUTION. $\frac{3}{4} = \frac{9}{12}$ and $\frac{5}{6} = \frac{10}{12}$; the ratio of 9 to 10 is $\frac{9}{10}$. ANS. $\frac{9}{10}$.

8. What is the ratio of $\frac{7}{8}$ to $\frac{9}{11}$? $\frac{7}{8}$ is what part of $\frac{9}{11}$?

9. What is the ratio of $\frac{9}{11}$ to $\frac{7}{8}$? $\frac{9}{11}$ is what part of $\frac{7}{8}$?

10. What is the ratio of 8 to 6? 8 is what part of 6?

11. If 6 yards of cloth cost 24 dollars, what will 8 yards of the same cloth cost? SOLUTION. The ratio of 8 to 6 is ⁴⁄₃, and ⁴⁄₃ of 24 dollars are 32 dollars. ANS. $32.

12. If 6 barrels of flour cost $56, what will 12 barrels cost?

13. If 12 barrels of molasses cost 132 dollars, what will 10 barrels cost?

14. If 6 men can do a piece of work in 48 days, in how many days can 12 men do the same work?

15. If 12 men can cut 48 cords of wood in one week, how many cords can 20 men cut in the same time?

16. If a staff, 3 feet long, cast a shadow 6 feet, what is the height of a steeple whose shadow at the same time is 150 feet?

17. What is the ratio of 6 to ½?

18. If a mechanic earns 6 shillings in ½ a day, how much can he earn in 6 days?

19. What is the ratio of 60 to 72? of 96 to 100? of 75 to 125?

20. If 24 men can build a wall in 60 days, how many men can build the same wall in 72 days? how many in 36 days?

21. A man walks 72 miles in 3 days, by walking 12 hours every day; in how many days could he walk the same distance, by walking 9 hours a day?

22. In what time can 7 men do a piece of work that 8 men can do in 15 days?

23. If 9 tons of hay will keep 8 horses 4 months, how many tons will keep 12 horses the same time?

24. If 6 barrels of flour cost 38 dollars, what will 8 barrels cost, at the same rate?

25. A farmer sold 14 pounds of butter, at 18 cents per pound, and received in payment sugar at 7 cents per pound; how many pounds of sugar did he receive?

26. How many bushels of potatoes, at 35 cents per bushel, will pay for 2 barrels of flour, at 3 dollars and 50 cents per barrel?

27. How many gallons of molasses, at 34 cents a gallon, can be bought for 12 dozen of eggs, at 17 cents a dozen?

28. Divide 14 into two such parts that one will be to the other as 3 to 4. SOLUTION. The ratio of 3 to 4 is $\frac{3}{4}$; $\frac{4}{4} + \frac{3}{4} = \frac{7}{4} = 14$; $\frac{4}{4} = 8$, and $\frac{3}{4} = 6$. ANS. 6 and 8.

29. Divide 24 into two such parts that one shall be to the other as 7 to 5.

30. Divide 32 into two such parts that one shall be to the other as 7 to 9.

31. Divide 40 into two such parts that one shall be to the other as 9 to 11.

32. Divide 39 into two such parts that one shall be to the other as 5 to 8.

33. Divide 36 into two such parts that one shall be to the other as 5 to 4.

34. Divide 36 into two such parts that one shall be three times the other.

35. Divide 42 into two such parts that one shall be six times the other.

36. A farmer had 96 sheep in two pastures. In one pasture there were three times as many sheep as in the other; how many were there in each?

37. A man bought a gold watch and chain for 160 dollars, paying 7 times as much for the watch as for the chain; what did he pay for each?

38. What is ratio?

39. What is the ratio of two numbers?

40. Why is not the ratio changed when we multiply or divide both terms by the same number?

41. What is proportion?

PARTNERSHIP.

SECTION XLIII.

PARTNERSHIP is the process of ascertaining the gain or loss of partners in trade. The money invested is called the *capital* or *stock*, and the profit to be divided is called the *dividend*. Each man shares the profit or loss according to his proportion of the stock.

1. A and B hired a pasture for 16 dollars. A put in 5 cows and B 3 cows; what ought each to pay? SOLUTION. 5 cows plus 3 cows are 8 cows. A, who puts in 5 cows, must pay ⅝ of 16 dollars, or 10 dollars; and B, who puts in 3 cows, must pay ⅜ of 16 dollars, or 6 dollars. ANS. A must pay 10 dollars, and B 6 dollars.

2. John and Henry bought a pair of pigeons, John paying 1 dollar, and Henry 75 cents; they afterwards sold them, and gained 35 cents; what was each boy's share of the gain?

3. A father divided 66 dollars between his two sons, giving to one 5 dollars as often as he gave to the other 6 dollars; what was each one's share?

4. Two men entered into partnership, and gained $50. The first man put in 4 times as much as the second; what was each one's share of the gain?

5. A, B and C, traded in company. A's stock was $1000, B's $1500, and C's $2000, and they gained $1800; what was each one's share of the gain?

OBS. A's gain was $\frac{1\,0}{4\,5}$ or $\frac{2}{9}$ of $1800.

6. A and B purchase a lot of land for $4500. A pays ⅓ of the price, and B the remainder. They gain by the sale of it 20 per cent. of the cost; what is each man's share of the gain?

7. Divide 27 cents between two boys, so that one of them shall have ⅘ as much as the other; what will each one receive?

8. Three men, A, B and C, perform a certain piece of work, for which they receive 96 dollars. A works 4 weeks, B works 3 weeks, and C works 1 week; what ought each to receive?

9. Two men, A and B, build a barn for 400 dollars. A employs 4 men 5 months, and B employs 5 men 6 months; what ought each one to receive?

OBS. 4 men for 5 months are the same as 20 men for 1 month.

10. Two men hire a pasture together. One man puts in 3 horses 4 months, and the other 6 horses 2 months; what part of the hire must each man pay?

11. A, B and C, traded in company; A put in $\frac{1}{2}$ of the stock, B put in $\frac{2}{5}$ of it, and C the remainder. They gained $3000; what was each man's share of the gain?

12. A man fails, owing to X $700, to Y $900, and to Z $400. His property amounts to $1500; how much will each creditor receive?

13. A man failing in business, owes to one of his creditors $1500, to another $2000, and to another $2500. His property amounts to $3000, which is divided among his creditors according to their respective demands. How much did each one receive?

14. Three men traded in company. A put in 2000 dollars, B put in 3000 dollars, and C put in 4000 dollars. They gained a sum equal to $\frac{2}{3}$ of the whole stock. What was each man's share of the gain?

15. A, B and C, receive $100 for a piece of work. A works 3 days a week, B 4 days, and C 5 days. What ought each one to receive as his share of the money?

16. What is partnership?

17. What is the money invested called?

18. What is the profit called?

19. How is the profit or loss shared?

CONTRACTIONS IN MULTIPLICATION.

SECTION XLIV.

THE following contractions may be studied as soon as the pupils are prepared to understand them. They have been arranged together, for convenient reference. The principle of each contraction should be fully shown to the pupils.

Any number containing two figures may be multiplied by 11, by arranging in decimal order the units' figure, the sum of the figures, and the figure in the tens' place; $63 \times 11 = 693$.

OBS. When the sum of the figures equals or exceeds ten, the left-hand figure must be added to the figure in the next place on the left. $75 \times 11 = 825$. SOLUTION. The units' figure is 5 ; the right-hand figure of the sum (12) is 2 ; the figure in the tens' place (7), increased by 1, is 8. ANS. 825.

1.	What is the product of 12×11 ?	13×11 ?
2.	What is the product of 14×11 ?	15×11 ?
3.	What is the product of 16×11 ?	17×11 ?
4.	What is the product of 18×11 ?	19×11 ?
5.	What is the product of 21×11 ?	22×11 ?
6.	What is the product of 23×11 ?	24×11 ?
7.	What is the product of 25×11 ?	26×11 ?
8.	What is the product of 27×11 ?	28×11 ?
9.	What is the product of 29×11 ?	31×11 ?
10.	What is the product of 32×11 ?	33×11 ?
11.	What is the product of 34×11 ?	35×11 ?
12.	What is the product of 36×11 ?	37×11 ?
13.	What is the product of 38×11 ?	39×11 ?
14.	What is the product of 41×11 ?	42×11 ?
15.	What is the product of 43×11 ?	44×11 ?
16.	What is the product of 45×11 ?	46×11 ?
17.	What is the product of 47×11 ?	48×11 ?
18.	What is the product of 49×11 ?	51×11 ?
19.	What is the product of 52×11 ?	53×11 ?
20.	What is the product of 54×11 ?	55×11 ?

21. What is the product of 56×11? 57×11?
22. What is the product of 58×11? 59×11?
23. What is the product of 61×11? 62×11?
24. What is the product of 63×11? 64×11?
25. What is the product of 65×11? 66×11?
26. What is the product of 67×11? 68×11?
27. What is the product of 69×11? 71×11?
28. What is the product of 72×11? 73×11?
29. What is the product of 74×11? 75×11?
30. What is the product of 76×11? 77×11?
31. What is the product of 78×11? 79×11?
32. What is the product of 81×11? 82×11?
33. What is the product of 83×11? 84×11?
34. What is the product of 85×11? 86×11?
35. What is the product of 87×11? 88×11?
36. What is the product of 89×11? 91×11?
37. What is the product of 92×11? 93×11?
38. What is the product of 94×11? 95×11?
39. What is the product of 96×11? 97×11?
40. What is the product of 98×11? 99×11?

41. What cost 54 pounds of crushed sugar, at 11 cents a pound?

42. If one yard of cotton cloth is worth 11 cents, what are 37 yards worth? 64 yards? 72 yards? 97 yards?

43. If a mechanic can earn 11 dollars in one week, how much can he earn in 27 weeks? in 45 weeks? in 87 weeks? in 76 weeks?

44. If one barrel of mackerel is worth 11 dollars, what are 23 barrels worth? 42 barrels? 67 barrels?

45. What is the cost of 29 pounds of Malaga raisins, at 11 cents a pound? What is the cost of 39 pounds, at the same rate? 53 pounds?

46. If a car run 11 miles in one hour, how many miles will it run in 65 hours? in 73 hours?

47. If one pound of beef cost 11 cents, what is the cost of 47 pounds? 59 pounds? 83 pounds?

SECTION XLV.

The product of any two numbers between 10 and 20 may be found by arranging in the decimal order, first, the right-hand figure of the product of the units, then the right-hand figure of the sum of the units, then the product of the tens.

Obs. In writing figures in the decimal order, whenever the product or sum contains more than one figure, the left-hand figure must be added to the figure in the next place on the left. $16 \times 17 = 272$. Solution. The right-hand figure of the product of the units ($6 \times 7 = 42$) is 2 ; the right-hand figure of the sum of the units ($6 + 7$) plus the tens of the product of the units (4) is 7 ; the product of the tens (1×1) plus 1 is 2.   Ans. 272.

 1. What is the product of 12×12? 13×12?
 2. What is the product of 14×12? 15×12?
 3. What is the product of 16×12? 17×12?
 4. What is the product of 18×12? 19×12?
 5. What is the product of 13×13? 14×13?
 6. What is the product of 15×13? 16×13?
 7. What is the product of 17×13? 18×13?
 8. What is the product of 19×13? 14×14?
 9. What is the product of 15×14? 16×14?
10. What is the product of 17×14? 18×14?
11. What is the product of 19×14? 15×15?
12. What is the product of 16×15? 17×15?
13. What is the product of 18×15? 19×15?
14. What is the product of 16×16? 17×16?
15. What is the product of 18×16? 19×16?
16. What is the product of 17×17? 18×17?
17. What is the product of 19×17? 18×18?
18. What is the product of 19×18? 19×19?

19. In one foot there are 12 inches; how many inches are there in 13 feet? in 16 feet? in 18 feet?

20. In one pound Troy there are 12 ounces; how

many ounces in 15 pounds? in 17 pounds? in 14 pounds?

21. In one pound Avoirdupois there are 16 ounces; how many ounces are there in 13 pounds? in 14 pounds? in 17 pounds? in 18 pounds? in 19 pounds?

22. How many square feet are there in a room 18 feet long and 16 feet wide? in a room 19 feet long, by 18 feet wide?

23. How many rods in a piece of land that is 18 rods long, and 17 rods wide?

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SECTION XLVI.

To find the product of any two numbers of two figures each, when the units' figure in each is 1.

Arrange in decimal order the unit figure (1), the sum of the tens, and the product of the tens. Thus: 61 × 71 = 4331. SOLUTION. The unit figure is 1; the sum of the tens (6 + 7) is 13; the product of the tens (6 × 7) plus the left-hand figure of the sum of the tens (1) is 43. ANS. 4331.

1. What is the product of 21 × 21?
2. What is the product of 31 × 21?
3. What is the product of 41 × 21?
4. What is the product of 51 × 21?
5. What is the product of 61 × 21?
6. What is the product of 71 × 21?
7. What is the product of 81 × 21?
8. What is the product of 91 × 21?
9. What is the product of 31 × 31?
10. What is the product of 41 × 31?
11. What is the product of 51 × 31?
12. What is the product of 61 × 31?

13. What is the product of 71 × 31 ?
14. What is the product of 81 × 31 ?
15. What is the product of 91 × 31 ?
16. What is the product of 41 × 41 ?
17. What is the product of 51 × 41 ?
18. What is the product of 61 × 41 ?
19. What is the product of 71 × 41 ?
20. What is the product of 81 × 41 ?
21. What is the product of 91 × 41 ?
22. What is the product of 51 × 51 ?
23. What is the product of 61 × 51 ?
24. What is the product of 71 × 51 ?
25. What is the product of 81 × 51 ?
26. What is the product of 91 × 51 ?
27. What is the product of 61 × 61 ?
28. What is the product of 71 × 61 ?
29. What is the product of 81 × 61 ?
30. What is the product of 91 × 61 ?
31. What is the product of 71 × 71 ?
32. What is the product of 81 × 71 ?
33. What is the product of 91 × 71 ?
34. What is the product of 81 × 81 ?
35. What is the product of 91 × 81 ?
36. What is the product of 91 × 91 ?

37. In one guinea there are 21 shillings; how many shillings are there in 21 guineas? in 31 guineas? in 41 guineas? in 51 guineas? in 61 guineas? in 71 guineas? in 81 guineas? in 91 guineas?

38. How many square rods are there in a piece of land 71 rods long, and 51 rods wide?

39. What will 61 bushels of potatoes cost, at 41 cents a bushel?

40. What will 51 pounds of butter cost, at 31 cents a pound?

41. What will 30½ pounds of sugar cost, at 10½ cents a pound?

SECTION XLVII.

To find the product of two or more figures by any multiple of 11.

By the figure denoting how many times 11 are contained in the multiplier, multiply first the units' figure of the multiplicand, then the sum of the units and tens, then the sum of the tens and hundreds, &c., and lastly the left-hand figure.   Thus, $73 \times 55 = 4015$.

SOLUTION. 11 are contained in 55, 5 times.   5 times 3 (the units' figure) $= 15$; the right-hand figure is 5 : 5 times 10 (7 $+$ 3, the sum of the units and tens) $= 50$; plus the one ten, are 51; the right-hand figure is 1 : 5 times 7 (the left-hand figure) $= 35$; plus the 5 from the 51 are 40.   ANS. 4015.

 1. What is the product of 24×22 ?
 2. What is the product of 48×22 ?
 3. What is the product of 67×22 ?
 4. What is the product of 84×22 ?
 5. What is the product of 27×33 ?
 6. What is the product of 47×33 ?
 7. What is the product of 64×33 ?
 8. What is the product of 84×33 ?
 9. What is the product of 29×44 ?
10. What is the product of 47×44 ?
11. What is the product of 62×44 ?
12. What is the product of 84×44 ?
13. What is the product of 29×55 ?
14. What is the product of 45×55 ?
15. What is the product of 67×55 ?
16. What is the product of 83×55 ?
17. What is the product of 27×66 ?
18. What is the product of 45×66 ?
19. What is the product of 67×66 ?
20. What is the product of 82×66 ?
21. What is the product of 24×77 ?

22. What is the product of   48 × 77 ?
23. What is the product of   67 × 77 ?
24. What is the product of   83 × 77 ?
25. What is the product of   29 × 88 ?
26. What is the product of   49 × 88 ?
27. What is the product of   68 × 88 ?
28. What is the product of   83 × 88 ?
29. What is the product of   22 × 99 ?
30. What is the product of   42 × 99 ?
31. What is the product of   62 × 99 ?
32. What is the product of   82 × 99 ?
33. What is the product of 124 × 66 ?

SOLUTION. 11 are contained in 66, 6 times. 6 times
4 are 24; the right-hand figure is 4.   The sum of the
units and tens is 6, and 6 times 6 are 36; plus 2 (from
the 24) are 38; the right-hand figure is 8.   The sum
of the tens and hundreds is 3, and 6 times 3 are 18;
plus three (from the 38) are 21; the right-hand figure
is 1.   6 times 1 (the left-hand figure) are 6; plus 2
(from the 21) are 8.   ANS. 8184.

OBS. If this section should prove too difficult for young
pupils, it may be omitted until they have finished those sec-
tions which are less difficult.

34. What is the product of 124 × 22 ?
35. What is the product of 396 × 22 ?
36. What is the product of 942 × 33 ?
37. What is the product of 116 × 88 ?
38. What is the product of 222 × 99 ?
39. What is the product of 328 × 44 ?
40. What is the product of 237 × 55 ?
41. What is the product of 144 × 38 ?
42. What is the product of 191 × 44 ?
43. What is the product of 181 × 55 ?
44. What is the product of 171 × 33 ?
45. What is the product of 161 × 22 ?
46. What is the product of 182 × 44 ?

SECTION XLVIII.

To find the product of two numbers, when the sum of the units is ten, and the preceding figure or figures are alike in each:

Multiply the preceding figure or figures of one of the numbers by the preceding figure or figures of the other, increased by one, and prefix the product to the product of the units.   When the product of the units is less than ten (10), a cipher must be written in the tens' place. $21 \times 29 = 609$.

SOLUTION. The preceding figure of one number (2), by the same figure increased by one $(2 + 1)$, is $= 6$; the product of the units (9×1) is 9 ; written with a cipher prefixed, 09. ANS. 609.

  1. What is the product of 21×29 ?
  2. What is the product of 22×28 ?
  3. What is the product of 23×27 ?
  4. What is the product of 24×26 ?
  5. What is the product of 25×25 ?
  6. What is the product of 31×39 ?
  7. What is the product of 32×38 ?
  8. What is the product of 37×33 ?
  9. What is the product of 38×32 ?
 10. What is the product of 39×31 ?
 11. What is the product of 41×49 ?
 12. What is the product of 42×48 ?
 13. What is the product of 43×47 ?
 14. What is the product of 44×46 ?
 15. What is the product of 45×45 ?
 16. What is the product of 51×59 ?
 17. What is the product of 52×58 ?
 18. What is the product of 53×57 ?
 19. What is the product of 54×56 ?
 20. What is the product of 55×55 ?
 21. What is the product of 61×69 ?

22. What is the product of 62 × 68 ?
23. What is the product of 63 × 67 ?
24. What is the product of 64 × 66 ?
25. What is the product of 65 × 65 ?
26. What is the product of 71 × 79 ?
27. What is the product of 72 × 78 ?
28. What is the product of 73 × 77 ?
29. What is the product of 74 × 76 ?
30. What is the product of 75 × 75 ?
31. What is the product of 81 × 89 ?
32. What is the product of 82 × 88 ?
33. What is the product of 83 × 87 ?
34. What is the product of 84 × 86 ?
35. What is the product of 85 × 85 ?
36. What is the product of 91 × 99 ?
37. What is the product of 92 × 98 ?
38. What is the product of 93 × 97 ?
39. What is the product of 94 × 96 ?
40. What is the product of 95 × 95 ?
41. What will cost 38 cows, at $32 apiece ?
42. What is the product of 184 × 186 ?

SOLUTION. The product of the units is 24. The product of the preceding figures of one number, multiplied by 1 more than the preceding figures of the other (18 × 19), is 342. ANS. 34224.

OBS. In multiplying 18 by 19, we must use the process shown in Section XLV. 8 × 9 = 72 ; 8 + 9 = 17, and 17 + 7 = 24 ; 2 + 1 = 3. ANS. 342.

43. What is the product of 123 × 127 ?
44. What is the product of 124 × 126 ?
45. What is the product of 125 × 125 ?
46. What is the product of 131 × 139 ?
47. What is the product of 132 × 138 ?
48. What is the product of 133 × 137 ?
49. What is the product of 134 × 136 ?
50. What is the product of 135 × 135 ?

51. What is the product of 141 × 149 ?
52. What is the product of 142 × 148 ?
53. What is the product of 143 × 147 ?
54. What is the product of 144 × 146 ?
55. What is the product of 145 × 145 ?
56. What is the product of 151 × 159 ?
57. What is the product of 152 × 158 ?
58. What is the product of 153 × 157 ?
59. What is the product of 154 × 156 ?
60. What is the product of 155 × 155 ?
61. What is the product of 161 × 169 ?
62. What is the product of 162 × 168 ?
63. What is the product of 163 × 167 ?
64. What is the product of 164 × 166 ?
65. What is the product of 165 × 165 ?
66. What is the product of 171 × 179 ?
67. What is the product of 172 × 178 ?
68. What is the product of 173 × 177 ?
69. What is the product of 174 × 176 ?
70. What is the product of 175 × 175 ?
71. What is the product of 181 × 189 ?
72. What is the product of 182 × 188 ?
73. What is the product of 183 × 187 ?
74. What is the product of 184 × 186 ?
75. What is the product of 185 × 185 ?
76. What is the product of 191 × 199 ?
77. What is the product of 192 × 198 ?
78. What is the product of 193 × 197 ?
79. What is the product of 194 × 196 ?
80. What is the product of 195 × 195 ?
81. What is the interest of $136 for 2 years, 2 months, and 24 days ?

SOLUTION.   The interest of 1 dollar for two years and 2 months (26 months) is 13 cents; for 24 days, it is 4 mills.   136 times 134 mills are 18224 mills.   ANS. $18.224.

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82. What is the interest of $124 for 2 years and 36 days?

83. What is the interest of $128 for 2 years and 12 days?

84. What is the interest of $125 for 2 years and 1 month?

85. What is the interest of $175 for 2 years and 11 months?

86. What is the interest of $195 for 3 years and 3 months?

87. A man bought a house-lot 189 feet long, and 181 feet wide; how many square feet were there?

88. The hammer of a clock strikes 156 times in 24 hours, or one day; how many times does it strike from the first of September to the first of February, inclusive?

89. If a car-wheel turn 188 times in a minute, how many times will it turn in 182 minutes?

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SECTION XLIX.

To find the product of two numbers containing two figures each, when the unit figures are alike, and the sum of the tens is 10:

Prefix the product of the tens, plus the units' figure of one of the numbers, to the product of the units. Thus $24 \times 84 = 2016$. When the product of the units is less than ten (10), a cipher must be written in the place of tens.

SOLUTION. The product of the tens (16) plus the unit figure (4) $= 20$; the product of the units (4×4) is 16. ANS. 2016.

1. What is the product of 22×82?
2. What is the product of 23×83?

3. What is the product of 24 × 84 ?
4. What is the product of 25 × 85 ?
5. What is the product of 26 × 86 ?
6. What is the product of 27 × 87 ?
7. What is the product of 28 × 88 ?
8. What is the product of 29 × 89 ?
9. What is the product of 32 × 72 ?
10. What is the product of 33 × 73 ?
11. What is the product of 34 × 74 ?
12. What is the product of 35 × 75 ?
13. What is the product of 36 × 76 ?
14. What is the product of 37 × 77 ?
15. What is the product of 38 × 78 ?
16. What is the product of 39 × 79 ?
17. What is the product of 42 × 62 ?
18. What is the product of 43 × 63 ?
19. What is the product of 44 × 64 ?
20. What is the product of 45 × 65 ?
21. What is the product of 46 × 66 ?
22. What is the product of 47 × 67 ?
23. What is the product of 48 × 68 ?
24. What is the product of 49 × 69 ?
25. What is the product of 52 × 52 ?
26. What is the product of 53 × 53 ?
27. What is the product of 54 × 54 ?
28. What is the product of 55 × 55 ?
29. What is the product of 56 × 56 ?
30. What is the product of 57 × 57 ?
31. What is the product of 58 × 58 ?
32. What is the product of 59 × 59 ?

33. What is the interest of $82 for 3 years and 8 months, at 6 per cent. ?

SOLUTION. The interest of one dollar for 3 years is 18 cents ; for 8 months, it is 4 cents ; 18 + 4 = 22. The interest of 82 dollars for 3 years and 8 months will be 82 times 22 cents, or 1804 cents. ANS. $18.04.

34. What is the interest of $77 for 6 years and 2 months?

35. What is the interest of $75 for 5 years and 10 months?

36. What is the interest of $87 for 3 years and 10 months?

37. What is the interest of $59 for 9 years and 10 months?

38. What is the interest of $49 for 11 years and 6 months?

,39. A drover bought 73 cows, at 33 dóllars apiece; what did they all cost him?

40. What will cost 46 bushels of corn, at 66 cents a bushel?

41. A grocer bought a hogshead of New Orleans molasses, containing 87 gallons, at 27 cents a gallon; what did it cost him?

42. At the rate of 36 miles an hour, how far will a railroad-car run in 76 hours?

43. In one day there are 24 hours; how many hours are there in 84 days?

44. If a wheel revolve 39 times in a second, how many times will it revolve in 79 seconds?

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SECTION L.

To find the product of two numbers, when the sum of the units is 10, and the difference of the figures preceding the units' figures is 1:

Prefix the square of the figure or figures preceding the units of the larger number, less 1, to the difference between the square of the units of the larger number and 100. Thus, 126 × 134 = 16884.

SOLUTION. The square of 13 (the figures preceding the units of the larger number) is 169; less 1 is 168.

The square of 4 (the units of the larger number) is 16; 100 — 16 = 84.   168 prefixed to 84 is 16884.

OBS. The product of a number multiplied into itself is called the square of that number.

1. What is the product of 21 × 39 ?
2. What is the product of 22 × 38 ?
3. What is the product of 23 × 37 ?
4. What is the product of 24 × 36 ?
5. What is the product of 25 × 35 ?
6. What is the product of 31 × 49 ?
7. What is the product of 32 × 48 ?
8. What is the product of 33 × 47 ?
9. What is the product of 34 × 46 ?
10. What is the product of 35 × 45 ?
11. What is the product of 26 × 34 ?
12. What is the product of 27 × 33 ?
13. What is the product of 28 × 32 ?
14. What is the product of 29 × 31 ?
15. What is the product of 36 × 44 ?
16. What is the product of 37 × 43 ?
17. What is the product of 38 × 42 ?
18. What is the product of 39 × 41 ?
19. What is the product of 41 × 59 ?
20. What is the product of 42 × 58 ?
21. What is the product of 43 × 57 ?
22. What is the product of 44 × 56 ?
23. What is the product of 45 × 55 ?
24. What is the product of 46 × 54 ?
25. What is the product of 47 × 53 ?
26. What is the product of 48 × 52 ?
27. What is the product of 49 × 51 ?
28. What is the product of 51 × 69 ?
29. What is the product of 52 × 68 ?
30. What is the product of 53 × 67 ?
31. What is the product of 54 × 66 ?
32. What is the product of 55 × 65 ?

33. What is the product of 56 × 64 ?
34. What is the product of 57 × 63 ?
35. What is the product of 58 × 62 ?
36. What is the product of 59 × 61 ?
37. What is the product of 61 × 79 ?
38. What is the product of 62 × 78 ?
39. What is the product of 63 × 77 ?
40. What is the product of 64 × 76 ?
41. What is the product of 65 × 75 ?
42. What is the product of 66 × 74 ?
43. What is the product of 67 × 73 ?
44. What is the product of 68 × 72 ?
45. What is the product of 69 × 71 ?
46. What is the product of 71 × 89 ?
47. What is the product of 72 × 88 ?
48. What is the product of 73 × 87 ?
49. What is the product of 74 × 86 ?
50. What is the product of 75 × 85 ?
51. What is the product of 76 × 84 ?
52. What is the product of 77 × 83 ?
53. What is the product of 78 × 82 ?
54. What is the product of 79 × 81 ?
55. What is the product of 91 × 109 ?
56. What is the product of 92 × 108 ?
57. What is the product of 93 × 107 ?
58. What is the product of 94 × 106 ?
59. What is the product of 96 × 104 ?
60. What is the product of 99 × 101 ?
61. What is the product of 101 × 119 ?
62. What is the product of 102 × 118 ?
63. What is the product of 103 × 117 ?
64. What is the product of 104 × 116 ?
65. What is the product of 106 × 114 ?
66. What is the product of 107 × 113 ?
67. What is the product of 108 × 112 ?
68. What is the product of 109 × 111 ?

69. What is the product of 111 × 129?
70. What is the product of 112 × 128?
71. What is the product of 113 × 127?
72. What is the product of 114 × 126?
73. What is the product of 116 × 124?
74. What is the product of 117 × 123?
75. What is the product of 118 × 122?
76. What is the product of 119 × 121?
77. What is the product of 122 × 138?
78. What is the product of 123 × 137?
79. What is the product of 124 × 136?
80. What is the product of 126 × 134?
81. What is the product of 127 × 133?
82. What is the product of 128 × 132?
83. What is the product of 129 × 131?
84. What is the product of 131 × 149?
85. What is the product of 132 × 148?
86. What is the product of 133 × 147?
87. What is the product of 134 × 146?
88. What is the product of 136 × 144?
89. What is the product of 137 × 143?
90. What is the product of 138 × 142?
91. What is the product of 139 × 141?
92. What is the product of 141 × 159?
93. What is the product of 142 × 158?
94. What is the product of 143 × 157?
95. What is the product of 144 × 156?
96. What is the product of 146 × 154?
97. What is the product of 147 × 153?
98. What is the product of 148 × 152?
99. What is the product of 149 × 151?

100. What is the interest of $153 for 2 years, 5 months, and 12 days, at 6 per cent.?

SOLUTION. The interest of one dollar for 2 years and 5 months (29 months) is 14 cents and 5 mills; for 12 days it is 2 mills; 145 mills plus 2 mills are 147

mills. The interest of $153 will be 153 times 147 mills. Ans. $22.491.

Obs. In finding the square of 15 in the above solution, we must use the process shown in Section XLV.

101. What is the interest of $142 for 2 years, 7 months, and 18 days?

102. What is the interest of $136 for 2 years and 24 days?

103. What is the interest of $181 for 2 years, 11 months, and 24 days?

104. How many square feet are there in a lot of land that is 146 feet long, and 134 feet wide?

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SECTION LI.

To find the product of two numbers containing two figures each, when the figures in the place of the units are alike:

Arrange, in the decimal order, first the right-hand figure of the product of the units; then the right-hand figure of the product of the sum of the tens, by the units' figure of one of the numbers; then the product of the tens. Thus, $27 \times 47 = 1269$.

Obs. When there are two figures in any sum or product, the left-hand figure must be added to the figure in the next place.

Solution. The product of the units (7×7) is 49; the right-hand figure is 9. The sum of the tens $(4 + 2)$, multiplied by the units (7), gives 42; plus 4 (from the 49) are 46; the right-hand figure is 6. The product of the tens (4×2) is 8; plus 4 (from the 46) are 12. Ans. 1269.

1. What is the product of 22 × 32 ?
2. What is the product of 22 × 42 ?
3. What is the product of 22 × 52 ?
4. What is the product of 22 × 62 ?
5. What is the product of 22 × 72 ?
6. What is the product of 22 × 82 ?
7. What is the product of 22 × 92 ?
8. What is the product of 23 × 33 ?
9. What is the product of 23 × 43 ?
10. What is the product of 23 × 53 ?
11. What is the product of 23 × 63 ?
12. What is the product of 23 × 73 ?
13. What is the product of 23 × 83 ?
14. What is the product of 23 × 93 ?
15. What is the product of 24 × 34 ?
16. What is the product of 24 × 44 ?
17. What is the product of 24 × 54 ?
18. What is the product of 24 × 64 ?
19. What is the product of 24 × 74 ?
20. What is the product of 24 × 84 ?
21. What is the product of 24 × 94 ?
22. What is the product of 26 × 36 ?
23. What is the product of 26 × 46 ?
24. What is the product of 26 × 56 ?
25. What is the product of 26 × 66 ?
26. What is the product of 26 × 76 ?
27. What is the product of 26 × 86 ?
28. What is the product of 26 × 96 ?
29. What is the product of 27 × 37 ?
30. What is the product of 27 × 47 ?
31. What is the product of 27 × 57 ?
32. What is the product of 27 × 67 ?
33. What is the product of 27 × 77 ?
34. What is the product of 27 × 87 ?
35. What is the product of 27 × 97 ?
36. What is the product of 28 × 38 ?

37. What is the product of 28 × 48 ?
38. What is the product of 28 × 58 ?
39. What is the product of 28 × 68 ?
40. What is the product of 28 × 78 ?
41. What is the product of 28 × 88 ?
42. What is the product of 28 × 98 ?
43. What is the product of 29 × 39 ?
44. What is the product of 29 × 49 ?
45. What is the product of 29 × 59 ?
46. What is the product of 29 × 69 ?
47. What is the product of 29 × 79 ?
48. What is the product of 29 × 89 ?
49. What is the product of 29 × 99 ?

50. In one day there are 24 hours; how many hours are there in 84 days ?

51. A provision dealer bought 65 pairs of chickens, at 65 cents a pair; what did they cost him ?

52. What is the interest of 67 dollars for 4 years and 6 months ?

53. What is the interest of 56 dollars for 4 years and 4 months ?

54. What is the interest of 49 dollars for 4 years and 10 months ?

55. How many square inches are there in a fire-place 5 feet 2 inches high, and 4 feet 4 inches wide ?

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SECTION LII.

To find the product of two numbers containing two figures each, when the figures in place of tens are alike :

Arrange in decimal order the right-hand figure of the product of the units, the right-hand figure of the product of the sum of the units by one of the tens, and then the product of the tens. Thus, 46 × 47 = 2162.

SOLUTION. The product of the units (7 × 6) is 42; the right-hand figure is 2.   The sum of the units (7 + 6), multiplied by the tens (4), gives 52; plus 4 (from the 42) are 56; the right-hand figure is 6.   The product of the tens (4 × 4) is 16; plus 5 (from the 56) are 21.   ANS. 2162.

1. What is the product of 21 × 28 ?
2. What is the product of 22 × 27 ?
3. What is the product of 23 × 26 ?
4. What is the product of 24 × 25 ?
5. What is the product of 25 × 26 ?
6. What is the product of 26 × 27 ?
7. What is the product of 27 × 28 ?
8. What is the product of 28 × 29 ?
9. What is the product of 29 × 29 ?
10. What is the product of 23 × 23 ?
11. What is the product of 27 × 27 ?
12. What is the product of 28 × 28 ?
13. What is the product of 24 × 24 ?
14. What is the product of 26 × 26 ?
15. What is the product of 33 × 33 ?
16. What is the product of 34 × 34 ?
17. What is the product of 36 × 36 ?
18. What is the product of 37 × 37 ?
19. What is the product of 38 × 38 ?
20. What is the product of 39 × 39 ?
21. What is the product of 31 × 38 ?
22. What is the product of 32 × 37 ?
23. What is the product of 33 × 36 ?
24. What is the product of 34 × 35 ?
25. What is the product of 35 × 36 ?
26. What is the product of 36 × 37 ?
27. What is the product of 37 × 38 ?
28. What is the product of 38 × 39 ?
29. What is the product of 42 × 42 ?
30. What is the product of 43 × 43 ?

31. What is the product of 44 × 44 ?
32. What is the product of 46 × 46 ?
33. What is the product of 47 × 47 ?
34. What is the product of 48 × 48 ?
35. What is the product of 49 × 49 ?
36. What is the product of 41 × 48 ?
37. What is the product of 42 × 47 ?
38. What is the product of 43 × 46 ?
39. What is the product of 44 × 45 ?
40. What is the product of 45 × 46 ?
41. What is the product of 46 × 47 ?
42. What is the product of 47 × 48 ?
43. What is the product of 62 × 62 ?
44. What is the product of 63 × 63 ?
45. What is the product of 64 × 64 ?
46. What is the product of 66 × 66 ?
47. What is the product of 67 × 67 ?
48. What is the product of 68 × 68 ?
49. What is the product of 72 × 72 ?
50. What is the product of 73 × 73 ?
51. What is the product of 74 × 74 ?
52. What is the product of 76 × 76 ?
53. What is the product of 77 × 77 ?
54. What is the product of 78 × 78 ?
55. What is the product of 79 × 79 ?
56. What is the product of 82 × 82 ?
57. What is the product of 83 × 83 ?
58. What is the product of 84 × 84 ?
59. What is the product of 86 × 86 ?
60. What is the product of 87 × 87 ?
61. What is the product of 88 × 88 ?
62. What is the product of 89 × 88 ?
63. What is the product of 92 × 92 ?
64. What is the product of 93 × 93 ?
65. What is the product of 96 × 96 ?
66. What is the product of 97 × 97 ?

SECTION LIII.

To find the product of two numbers containing two figures each, when neither the unit figures nor the figures in the place of the tens are alike :

Arrange in decimal order first the product of the units ; then the product of the tens of the first, by the units of the second, plus the product of the units of the first by the tens of the second; then the product of the tens. Thus, $24 \times 35 = 840$.

SOLUTION. The product of the units is 20. The product of the tens of the first by the units of the second ($2 \times 5 = 10$), plus the product of the units of the first by the tens of the second ($4 \times 3 = 12$), are 22 ; plus 2 (from the 20) are 24. The product of the tens ($3 \times 2 = 6$) plus 2 (from the 24) are 8. ANS. 840.

1. What is the product of 24×33 ?
2. What is the product of 25×34 ?
3. What is the product of 26×35 ?
4. What is the product of 27×36 ?
5. What is the product of 28×37 ?
6. What is the product of 29×38 ?
7. What is the product of 32×43 ?
8. What is the product of 33×45 ?
9. What is the product of 34×46 ?
10. What is the product of 35×47 ?
11. What is the product of 36×48 ?
12. What is the product of 37×49 ?
13. What is the product of 38×43 ?
14. What is the product of 39×44 ?
15. What is the product of 45×53 ?
16. What is the product of 46×52 ?
17. What is the product of 47×56 ?
18. What is the product of 48×57 ?

19. What is the product of 49 × 58 ?
20. What is the product of 52 × 63 ?
21. What is the product of 53 × 64 ?
22. What is the product of 54 × 65 ?
23. What is the product of 56 × 67 ?
24. What is the product of 57 × 38 ?
25. What is the product of 58 × 39 ?
26. What is the product of 59 × 27 ?
27. What is the product of 63 × 24 ?
28. What is the product of 64 × 35 ?
29. What is the product of 65 × 37 ?
30. What is the product of 66 × 28 ?
31. What is the product of 67 × 34 ?
32. What is the product of 68 × 35 ?
33. What is the product of 69 × 36 ?
34. What is the product of 72 × 45 ?
35. What is the product of 73 × 44 ?
36. What is the product of 74 × 46 ?
37. What is the product of 76 × 53 ?
38. What is the product of 77 × 54 ?
39. What is the product of 78 × 65 ?
40. What is the product of 73 × 54 ?
41. What is the product of 79 × 63 ?
42. What is the product of 74 × 57 ?

43. A boy reads 56 pages of history every day; how many pages will he read in 84 days ?

44. How many pounds of coffee are there in 38 bags, each one of which contains 45 pounds ?

45. At the Brighton cattle-market the sheep were kept in 22 different pens, averaging 48 sheep to each pen ; how many sheep were there at the market on that day ?

46. What is the interest of $49 for 8 years and 10 months ?

47. What is the interest of $53 for 6 years and 2 months ?

SECTION LIV.

To find the product of any two mixed numbers whose fractional parts are halves:

Prefix the product of the whole numbers, plus one half their sum, to the fraction $\frac{1}{4}$. Thus, $3\frac{1}{2} \times 3\frac{1}{2} = 12\frac{1}{4}$.

SOLUTION. $3 \times 3 = 9$; one half of $(3 + 3) = 3$; and $9 + 3 = 12$. 12 prefixed to $\frac{1}{4}$ is $12\frac{1}{4}$. ANS. $12\frac{1}{4}$.

When the sum of the numbers is not an even number, add one half *the next smaller* number to the product, and prefix the sum to $\frac{3}{4}$. Thus, $2\frac{1}{2} \times 3\frac{1}{2} = 8\frac{3}{4}$.

SOLUTION. $2 \times 3 = 6$. The sum of the numbers is 5; and one half of 4 is 2. $6 + 2 = 8$, and 8 prefixed to $\frac{3}{4}$ is $8\frac{3}{4}$. ANS. $8\frac{3}{4}$.

1. What is the product of $2\frac{1}{2} \times 4\frac{1}{2}$?
2. What is the product of $2\frac{1}{2} \times 5\frac{1}{2}$?
3. What is the product of $2\frac{1}{2} \times 6\frac{1}{2}$?
4. What is the product of $2\frac{1}{2} \times 10\frac{1}{2}$?
5. What is the product of $2\frac{1}{2} \times 18\frac{1}{2}$?
6. What is the product of $3\frac{1}{2} \times 4\frac{1}{2}$?
7. What is the product of $3\frac{1}{2} \times 6\frac{1}{2}$?
8. What is the product of $3\frac{1}{2} \times 9\frac{1}{2}$?
9. What is the product of $3\frac{1}{2} \times 10\frac{1}{2}$?
10. What is the product of $13\frac{1}{2} \times 18\frac{1}{2}$?
11. What is the product of $4\frac{1}{2} \times 5\frac{1}{2}$?
12. What is the product of $4\frac{1}{2} \times 6\frac{1}{2}$?
13. What is the product of $4\frac{1}{2} \times 8\frac{1}{2}$?
14. What is the product of $4\frac{1}{2} \times 11\frac{1}{2}$?
15. What is the product of $4\frac{1}{2} \times 17\frac{1}{2}$?
16. What is the product of $5\frac{1}{2} \times 5\frac{1}{2}$?
17. What is the product of $5\frac{1}{2} \times 8\frac{1}{2}$?
18. What is the product of $5\frac{1}{2} \times 12\frac{1}{2}$?
19. What is the product of $5\frac{1}{2} \times 15\frac{1}{2}$?
20. What is the product of $5\frac{1}{2} \times 19\frac{1}{2}$?

21. What is the product of $6\frac{1}{2} \times 8\frac{1}{2}$?
22. What is the product of $6\frac{1}{2} \times 10\frac{1}{2}$?
23. What is the product of $6\frac{1}{2} \times 13\frac{1}{2}$?
24. What is the product of $6\frac{1}{2} \times 15\frac{1}{2}$?
25. What is the product of $6\frac{1}{2} \times 18\frac{1}{2}$?
26. What is the product of $7\frac{1}{2} \times 8\frac{1}{2}$?
27. What is the product of $7\frac{1}{2} \times 9\frac{1}{2}$?
28. What is the product of $7\frac{1}{2} \times 11\frac{1}{2}$?
29. What is the product of $7\frac{1}{2} \times 13\frac{1}{2}$?
30. What is the product of $7\frac{1}{2} \times 15\frac{1}{2}$?
31. What is the product of $8\frac{1}{2} \times 8\frac{1}{2}$?
32. What is the product of $8\frac{1}{2} \times 9\frac{1}{2}$?
33. What is the product of $8\frac{1}{2} \times 11\frac{1}{2}$?
34. What is the product of $8\frac{1}{2} \times 12\frac{1}{2}$?
35. What is the product of $8\frac{1}{2} \times 15\frac{1}{2}$?
36. What is the product of $9\frac{1}{2} \times 9\frac{1}{2}$?
37. What is the product of $9\frac{1}{2} \times 10\frac{1}{2}$?
38. What is the product of $9\frac{1}{2} \times 12\frac{1}{2}$?
39. What is the product of $9\frac{1}{2} \times 15\frac{1}{2}$?
40. What is the product of $9\frac{1}{2} \times 17\frac{1}{2}$?
41. What is the product of $10\frac{1}{2} \times 10\frac{1}{2}$?
42. What is the product of $10\frac{1}{2} \times 15\frac{1}{2}$?
43. What is the product of $10\frac{1}{2} \times 18\frac{1}{2}$?
44. What is the product of $10\frac{1}{2} \times 19\frac{1}{2}$?
45. What is the product of $10\frac{1}{2} \times 20\frac{1}{2}$?

46. What will cost $16\frac{1}{2}$ pounds of raisins, at $12\frac{1}{2}$ cents a pound ?

47. If a man walk $3\frac{1}{2}$ miles an hour, how many miles can he walk in $10\frac{1}{2}$ hours ?

48. If a man walk $4\frac{1}{2}$ miles an hour, how many miles can he walk in $10\frac{1}{2}$ hours ?

49. What is the interest of $40\frac{1}{2}$ dollars for 1 year and 9 months ?

50. How many square inches in a slate $11\frac{1}{2}$ inches long and $8\frac{1}{2}$ inches wide ?

SECTION LV.

To find the product of two mixed numbers, when the whole numbers are alike, and the sum of the fractions is 1 :

Multiply one whole number by the other increased by 1, and prefix the product to the product of the fractions. Thus, $6\frac{2}{7} \times 6\frac{5}{7} = 42\frac{6}{25}$.

SOLUTION. 6 multiplied by $6 + 1$ (7) is 42, prefixed to $\frac{2}{5} \times \frac{3}{5}$ ($\frac{6}{25}$), is $42\frac{6}{25}$.

1. What is the product of $4\frac{1}{4} \times 4\frac{3}{4}$?
2. What is the product of $4\frac{1}{5} \times 4\frac{4}{5}$?
3. What is the product of $4\frac{3}{6} \times 4\frac{3}{6}$?
4. What is the product of $4\frac{3}{7} \times 4\frac{4}{7}$?
5. What is the product of $4\frac{4}{9} \times 4\frac{5}{9}$?
6. What is the product of $5\frac{2}{4} \times 5\frac{2}{4}$?
7. What is the product of $5\frac{2}{5} \times 5\frac{3}{5}$?
8. What is the product of $5\frac{3}{7} \times 5\frac{4}{7}$?
9. What is the product of $5\frac{1}{11} \times 5\frac{10}{11}$?
10. What is the product of $5\frac{2}{12} \times 5\frac{10}{12}$?
11. What is the product of $6\frac{1}{2} \times 6\frac{1}{2}$?
12. What is the product of $6\frac{3}{8} \times 6\frac{5}{8}$?
13. What is the product of $6\frac{1}{7} \times 6\frac{6}{7}$?
14. What is the product of $6\frac{1}{12} \times 6\frac{11}{12}$?
15. What is the product of $6\frac{1}{13} \times 6\frac{12}{13}$?
16. What is the product of $7\frac{3}{8} \times 7\frac{5}{8}$?
17. What is the product of $7\frac{1}{9} \times 7\frac{8}{9}$?
18. What is the product of $7\frac{1}{9} \times 7\frac{8}{9}$?
19. What is the product of $7\frac{2}{10} \times 7\frac{8}{10}$?
20. What is the product of $7\frac{2}{11} \times 7\frac{9}{11}$?
21. What is the product of $8\frac{3}{8} \times 8\frac{5}{8}$?
22. What is the product of $8\frac{7}{10} \times 8\frac{3}{10}$?
23. What is the product of $8\frac{3}{11} \times 8\frac{8}{11}$?
24. What is the product of $8\frac{4}{12} \times 8\frac{8}{12}$?
25. What is the product of $8\frac{2}{13} \times 8\frac{11}{13}$?

26. What is the product of $9\frac{4}{8} \times 9\frac{4}{8}$?

27. What is the product of $9\frac{7}{9} \times 9\frac{2}{9}$?

28. What is the product of $9\frac{5}{10} \times 9\frac{4}{10}$?

29. What is the product of $9\frac{4}{11} \times 9\frac{7}{11}$?

30. What is the product of $9\frac{7}{12} \times 9\frac{5}{12}$?

31. What is the product of $10\frac{6}{8} \times 10\frac{3}{8}$?

32. What is the product of $10\frac{5}{10} \times 10\frac{5}{10}$?

33. What is the product of $10\frac{6}{11} \times 10\frac{5}{11}$?

34. What is the product of $10\frac{6}{12} \times 10\frac{6}{12}$?

35. What is the product of $10\frac{10}{13} \times 10\frac{3}{13}$?

36. Pearl ashes are worth $5\frac{7}{8}$ cents per pound ; what is the cost of $5\frac{1}{8}$ pounds, at that rate ?

37. Coal from Pictou is sold for $6\frac{1}{2}$ dollars a chaldron (36 bushels) ; what is paid for $6\frac{1}{2}$ chaldrons ?

38. Cannel coal is sold for $12\frac{1}{4}$ dollars a chaldron ; what is paid for $12\frac{1}{4}$ chaldrons of Cannel coal ?

39. A grocer sells Manilla coffee, in packages of $10\frac{5}{8}$ pounds each, for $10\frac{5}{8}$ cents per pound ; what does he receive for a package of the coffee ?

40. Logwood from St. Domingo is sold in Boston for $14\frac{3}{4}$ dollar a ton (2240 pounds) ; what is received for $14\frac{1}{4}$ tons of the logwood ?

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SECTION LVI.

To find the square of any mixed number, whose fractional part is $\frac{1}{4}$:

OBS. The product of a number multiplied into itself is called the square of the number.

To the square of the whole number, when even, add $\frac{1}{2}$ of itself, and prefix the sum to $\frac{1}{16}$; when it is not even, add to the square of the whole number $\frac{1}{2}$ of the next smaller number, and prefix the sum to $\frac{9}{16}$. Thus, the square of $4\frac{1}{4}$ is $18\frac{1}{16}$. The square of 4 is 16 ; $\frac{1}{2}$ of 4 is 2 ; $16 + 2 = 18$; 18 prefixed to $\frac{1}{16}$ is $18\frac{1}{16}$.

The square of $3\frac{1}{4}$ is $10\frac{9}{16}$.   The square of 3 is 9; $\frac{1}{2}$ of 2, the next smaller number, is 1; $9 + 1$ is 10; 10 prefixed to $\frac{9}{16}$ is $10\frac{9}{16}$.

1. What is the square of $2\frac{1}{4}$?   of $5\frac{1}{4}$?
2. What is the square of $7\frac{1}{4}$?   of $8\frac{1}{4}$?
3. What is the square of $10\frac{1}{4}$?   of $11\frac{1}{4}$?
4. What is the square of $13\frac{1}{4}$?   of $14\frac{1}{4}$?
5. What is the square of $16\frac{1}{4}$?   of $17\frac{1}{4}$?
6. What is the square of $19\frac{1}{4}$?   of $20\frac{1}{4}$?
7. What is the square of $25\frac{1}{4}$?   of $35\frac{1}{4}$?
8. What is the square of $40\frac{1}{4}$?   of $50\frac{1}{4}$?
9. What is the square of $6\frac{1}{4}$?   of $9\frac{1}{4}$?
10. What is the square of $12\frac{1}{4}$?   of $15\frac{1}{4}$?
11. What is the square of $18\frac{1}{4}$?   of $30\frac{1}{4}$?
12. What is the square of $45\frac{1}{4}$?   of $60\frac{1}{4}$?

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SECTION LVII.

To find the product of any two mixed numbers, when the difference of the whole numbers is 1, and the sum of the fractional parts is 1:

Prefix the square of the larger whole number, less 1, to the difference of the square of the fraction of the larger number and 1.   Thus, $5\frac{1}{6} \times 6\frac{5}{6} = 35\frac{11}{36}$.

SOLUTION. The square of 6 is 36, less 1, is 35. The square of $\frac{5}{6}$ is $\frac{25}{36}$, subtracted from $1 = \frac{11}{36}$. ANS. $35\frac{11}{36}$.

1. What is the product of $2\frac{1}{3} \times 3\frac{2}{3}$?
2. What is the product of $3\frac{2}{5} \times 4\frac{3}{5}$?
3. What is the product of $4\frac{3}{7} \times 5\frac{4}{7}$?
4. What is the product of $5\frac{5}{8} \times 6\frac{1}{2}$?
5. What is the product of $6\frac{2}{7} \times 7\frac{1}{2}$?
6. What is the product of $7\frac{3}{8} \times 8\frac{5}{8}$?
7. What is the product of $7\frac{1}{4} \times 8\frac{1}{2}$?
8. What is the product of $7\frac{3}{11} \times 8\frac{8}{11}$?

9. What is the product of $7\frac{1}{3} \times 8\frac{7}{8}$?
10. What is the product of $8\frac{4}{11} \times 9\frac{7}{11}$?
11. What is the product of $9\frac{2}{13} \times 10\frac{1}{11}$?
12. What is the product of $8\frac{2}{3} \times 9\frac{1}{4}$?
13. What is the product of $8\frac{2}{5} \times 9\frac{2}{3}$?
14. What is the product of $8\frac{4}{7} \times 9\frac{3}{4}$?
15. What is the product of $9\frac{2}{7} \times 10\frac{1}{2}$?
16. What is the product of $12\frac{1}{5} \times 13\frac{1}{4}$?
17. What is the product of $13\frac{1}{4} \times 14\frac{2}{5}$?
18. What is the product of $14\frac{1}{4} \times 15\frac{1}{4}$?
19. What is the product of $14\frac{2}{3} \times 15\frac{1}{5}$?
20. What is the product of $12\frac{2}{5} \times 13\frac{2}{5}$?
21. What is the product of $14\frac{2}{3} \times 15\frac{1}{5}$?
22. What is the product of $16\frac{2}{3} \times 17\frac{1}{3}$?
23. What is the product of $17\frac{2}{3} \times 18\frac{2}{3}$?
24. What is the product of $19\frac{1}{3} \times 20\frac{2}{3}$?

25. What cost $7\frac{3}{4}$ yards of cloth, at $6\frac{1}{4}$ cents a yard?
26. What cost $9\frac{3}{4}$ yards of cloth, at $8\frac{1}{4}$ cents a yard?
27. What cost $17\frac{1}{2}$ pounds of butter, at $16\frac{3}{4}$ cents per pound?
28. What cost $19\frac{1}{4}$ pounds of butter, at $18\frac{3}{4}$ cents per pound?

SECTION LVIII.

To find the square of any mixed number, when the fractional part is $\frac{3}{4}$:

Prefix the square of the whole number, plus $\frac{3}{4}$ of itself, to the square of $\frac{3}{4}$. Thus, $6\frac{3}{4} \times 6\frac{3}{4} = 45\frac{9}{16}$.

SOLUTION. The square of 6 (36) plus $\frac{3}{4}$ of 6 (9) are 45. The square of $\frac{3}{4}$ is $\frac{9}{16}$. ANS. $45\frac{9}{16}$.

When the whole number is not even, prefix the square of the whole number, plus $\frac{3}{4}$ of the next smaller number, increased by 2, to $\frac{1}{16}$. Thus, $5\frac{3}{4} \times 5\frac{3}{4} = 33\frac{1}{16}$.

SOLUTION. The square of 5 (25) plus $\frac{3}{4}$ of 4 (6) are 31: $31 + 2 = 33$. ANS. $33\frac{1}{16}$.

1. What is the square of $2\frac{3}{4}$?
2. What is the square of $3\frac{3}{4}$?
3. What is the square of $4\frac{3}{4}$?
4. What is the square of $5\frac{3}{4}$?
5. What is the square of $6\frac{3}{4}$?
6. What is the square of $7\frac{3}{4}$?
7. What is the square of $8\frac{3}{4}$?
8. What is the square of $9\frac{3}{4}$?
9. What is the square of $10\frac{3}{4}$?
10. What is the square of $11\frac{3}{4}$?
11. What is the square of $12\frac{3}{4}$?
12. What is the square of $13\frac{3}{4}$?
13. What is the square of $14\frac{3}{4}$?
14. What is the square of $15\frac{3}{4}$?
15. What is the square of $16\frac{3}{4}$?
16. What is the square of $17\frac{3}{4}$?
17. What is the square of $18\frac{3}{4}$?
18. What is the square of $19\frac{3}{4}$?
19. What is the square of $25\frac{3}{4}$?
20. What is the square of $35\frac{3}{4}$?
21. What is the square of $31\frac{3}{4}$?
22. What is the square of $41\frac{3}{4}$?
23. What is the square of $75\frac{3}{4}$?
24. What is the square of $85\frac{3}{4}$?
25. What is the square of $95\frac{3}{4}$?

SECTION LIX.

To find the product of any two numbers ending in 5:

Prefix the product of the figures preceding the 5 in each number, plus $\frac{1}{2}$ their sum, to 25. Thus, $185 \times 45 = 8325$. SOLUTION. The product of the figures preceding the 5 in the larger number (18) by the figure preceding the 5 in the smaller (4) is 72; plus half their sum $(18 + 4)$, $11 = 83$; and 83 prefixed to 25 is 8325. ANS. 8325.

When the sum of the preceding figures is an odd number, add to the product ½ of the next smaller number, and prefix the sum to 75. Thus, 55 × 25 = 1375.

SOLUTION. 5 × 2 = 10 ; ½ of 6 (next smaller number to 7) is 3 ; 10 + 3 = 13, and this prefixed to 75 is 1375. ANS. 1375.

1. What is the product of 25 × 25 ?
2. What is the product of 75 × 25 ?
3. What is the product of 125 × 25 ?
4. What is the product of 115 × 35 ?
5. What is the product of 145 × 35 ?
6. What is the product of 165 × 35 ?
7. What is the product of 195 × 35 ?
8. What is the product of 205 × 35 ?
9. What is the product of 75 × 45 ?
10. What is the product of 85 × 45 ?
11. What is the product of 115 × 45 ?
12. What is the product of 25 × 55 ?
13. What is the product of 95 × 55 ?
14. What is the product of 125 × 55 ?
15. What is the product of 195 × 55 ?
16. What is the product of 75 × 65 ?
17. What is the product of 95 × 65 ?
18. What is the product of 105 × 65 ?
19. What is the product of 155 × 65 ?
20. What is the product of 85 × 75 ?
21. What is the product of 135 × 75 ?
22. What is the product of 135 × 85 ?
23. What is the product of 145 × 85 ?
24. What is the product of 145 × 95 ?
25. What is the product of 205 × 95 ?

26. At 75 cents a bushel, what will 35 bushels of corn cost ?

27. What will 35 bushels of oats cost, at 45 cents a bushel ?

28. If a man earn 125 cents in one day, how much will he earn in 25 days ?

SECTION LX.

To find the square of any number ending in 25:

To the square of the figure or figures preceding the 25, add one half as many units as there are in the number that is squared, and to this sum annex 0625 (the square of 25 with a 0 prefixed).

If the figure or figures preceding the 25 are odd, add to the square one half of the next smaller number, and to this sum annex 5625 (the square of 75). Thus the square of 925 is 855625.

SOLUTION. The square of 9 is 81; plus one half the next smaller number (8) are 85; and 85 with 5625 annexed is 855625.

  1. What is the square of 225 ?
  2. What is the square of 325 ?
  3. What is the square of 425 ?
  4. What is the square of 525 ?
  5. What is the square of 625 ?
  6. What is the square of 725 ?
  7. What is the square of 825 ?
  8. What is the square of 925 ?
  9. What is the square of 1025 ?
10. What is the square of 1125 ?
11. What is the square of 1225 ?
12. What is the square of 1325 ?
13. What is the square of 1425 ?
14. What is the square of 1525 ?
15. What is the square of 1625 ?
16. What is the square of 1725 ?
17. What is the square of 1825 ?
18. What is the square of 1925 ?
19. What is the square of 2025 ?
20. What is the square of 3025 ?
21. What is the square of 4025 ?

SECTION LXI.

To find the product of two numbers one of which ends in 75 and the other in 25, when the preceding figure or figures in each are alike.

Multiply the preceding figure or figures of one number by the same increased by 1, and to the product annex 1875. Thus, 825 × 875 = 721875.

SOLUTION. The figure preceding the 75 (8) multiplied by the same increased by 1 (9) gives 72; 72 with 1875 annexed is 721875. ANS. 721875.

1. What is the product of   225 × 275 ?
2. What is the product of   325 × 375 ?
3. What is the product of   425 × 475 ?
4. What is the product of   525 × 575 ?
5. What is the product of   625 × 675 ?
6. What is the product of   725 × 775 ?
7. What is the product of   825 × 875 ?
8. What is the product of   925 × 975 ?
9. What is the product of 1025 × 1075 ?
10. What is the product of 1125 × 1175 ?
11. What is the product of 1225 × 1275 ?
12. What is the product of 1325 × 1375 ?
13. What is the product of 1425 × 1475 ?
14. What is the product of 1525 × 1575 ?
15. What is the product of 1625 × 1675 ?

OBS. In multiplying 16 by 17, the pupil must use the process given in Section XLV.

16. What is the product of 1725 × 1775 ?
17. What is the product of 1825 × 1875 ?
18. What is the product of 1925 × 1975 ?
19. What is the product of 2025 × 2075 ?
20. What is the product of 2125 × 2175 ?

OBS. In multiplying 21 by 22, the pupil must use the process shown in Section LIII.

21. What is the product of ·3125 × 3175 ?
22. What is the product of 4125 × 4175 ?
23. What is the product of 5125 × 5175 ?
24. What will cost 175 young fruit trees, at $1.25 apiece ?
25. What is the interest of 375 dollars for 5 years and 5 months, at 6 per cent. ?
26. What is the interest of 225 dollars for 4 years and 7 months, at 6 per cent. ?

SECTION LXII.

To find the square of any number ending in 75 :

Prefix the square of the figures preceding 75, plus ⅔ of these figures, to 5625 (the square of 75). Thus, the square of 1275 is 1625625.

SOLUTION. The square of 12 is 144; ⅔ of 12 are 18; 144 + 18 = 162; and this prefixed to 5625 is 1625625.

When the figures preceding the 75 are not an even number, add to the square ⅔ of the next smaller number, plus 2, and annex to the sum 0625. Thus, the square of 375 is 140625.

SOLUTION. The square of 3 is 9; ⅔ of 2 are 3; 9 + 3 + 2 = 14; to which annex 0625, and the answer will be 140625.

1. What is the square of 275 ?
2. What is the square of 375 ?
3. What is the square of 475 ?
4. What is the square of 575 ?
5. What is the square of 675 ? ·
6. What is the square of 775 ?
7. What is the square of 875 ?
8. What is the square of 975 ?

9. What is the square of 1075?
10. What is the square of 1175?
11. What is the square of 1275?
12. What is the square of 1375?
13. What is the square of 1475?
14. What is the square of 1575?
15. What is the square of 2075?

Obs. The pupil must fix firmly in his mind the figures to be annexed, and then the above questions can be readily performed.

SECTION LXIII.

MISCELLANEOUS QUESTIONS.

1. At what part of their cost must goods be sold, to gain 20 per cent.?

2. Reduce to common fractions the following: 10 per cent. **Ans.** $\frac{1}{10}$. 12½ per cent. **Ans.** $\frac{1}{8}$. 16⅔ per cent. 20 per cent. 25 per cent. 33⅓ per cent. 37½ per cent. 50 per cent.

3. At what part of their original cost must goods be sold, to gain 12½ per cent.? 33⅓ per cent.? 37½ per cent.?

4. A grocer bought flour at 5 dollars, but is obliged to sell it at a loss of 20 per cent.; at what price per barrel does he sell it?

5. A man bought cloth at 6 dollars and sold it at 7 dollars each yard; what per cent. did he gain?

6. What is the interest of 75 dollars for 15 days? for 12 days? for 10 days?

Obs. The interest for 60 days being as many cents as there are dollars in the principal, the interest for 15 days will be $\frac{15}{60}$ or $\frac{1}{4}$ as many cents as there are dollars in the principal.

7. What is the interest of $120 for 20 days?

8. What is the interest of $60 for 30 days?

9. What is the interest of $120 for 19 (15 + 4) days ?

10. What is the interest of $270 for 22 days ?

11. Divide 38 into two parts which shall be to each other as ⅝ to ¾.

12. A person being asked the hour of the day, said that the time past noon was ⅔ of the time to midnight; what was the time ?

13. What is the number to which if ¾ and ⅖ of itself be added the sum will be 43 ?

14. John is twice as old as Mary, and she is four times as old as George. The sum of all their ages is 39 ; what is the age of each ?

15. A gentleman gave $350 for his horse and chaise, but the horse was worth only ⅖ as much as the chaise ; what was the cost of each ?

16. One fourth of a man's age, plus 5 years, is equal to one third of his age ; how old is he ?

17. How many times in 12 hours are the minute and hour hands of a clock together ? At what time between 2 and 3 o'clock ? (₂⁄₁₁ of 12.)

18. A, B and C, shared $148 so that A had $12 more than B, and $20 more than C ; how much had each ?

19. A, B and C, shared $180 so that A had twice as much as B, and C had twice as much as A and B both ; how many dollars had each ?

20. Divide $159 between two persons so that ⅘ of what the first receives shall be equal to ⁵⁄₇ of what the second receives ; what will you give to each ?

Obs. The ratio of ⅘ to ⁵⁄₇ == ⅘ ÷ ⁵⁄₇ == ²⁸⁄₂₅, or 28 to 25. One must have ⅖⅞₃, and the other ²⁵⁄₅₃.

21. Divide $190 into two such parts that ¾ of one shall be equal to ⅖ of the other.

22. At what time between 7 and 8 are the hour and minute hands of the clock together ?

23. At what time after 5 o'clock are the hands of the clock first opposite each other? ($\frac{5}{11}$ or $\frac{11}{22}$ of 12.)

24. What o'clock is it now in London? In St. Petersburg? In Constantinople? In New Orleans? In St Louis?

25. A man being asked the hour, said that $\frac{1}{3}$ of the time from midnight was equal to $\frac{3}{8}$ of the time to noon; what was the hour?

26. A farmer has his sheep in four pastures. In the first pasture he has $\frac{1}{3}$ of his flock, in the second he has $\frac{1}{4}$ of his flock, in the third he has $\frac{1}{6}$ of his flock, and in the fourth he has 13 sheep; how many sheep has he in all?

27. A hare starts 60 rods before a greyhound, but the greyhound runs 9 rods while the hare is running 6 rods; how many rods must the greyhound run to overtake the hare?

28. A man sells wood for $5.50 a cord, which is 10 per cent. more than it cost him; how much did it cost him?

29. At what time between 4 and 5 are the hands of the clock exactly opposite? At what time between 9 and 10?

30. A man sells coal at $8 a ton, thereby gaining 20 per cent.; what did the coal cost him?

31. If cloth cost 4 dollars a yard, at what price must it be sold to gain 25 per cent.?

32. If sugar cost 10 cents a pound, at what price must it be sold to lose 10 per cent.?

33. A merchant bought molasses at 40 cents a gallon, and sold it for 50 cents; what did he gain per cent.?

34. A gentleman paid $\frac{1}{2}$ of his ready money for garden tools, $\frac{1}{4}$ of it for seeds, $\frac{1}{6}$ of it for ploughing, and had 18 dollars left to pay for planting; how much money had he at first?

35. One half, one third and one seventh of a certain number, plus 18, are 100; what is the number?

36. Charles is three times as old as Edward, and Edward is one half as old as James. The difference of age between Charles and James is 5 years; how old is each?

37. William is 8 years older than his sister Jane, and Jane's age is equal to $\frac{1}{4}$ of William's; what is the age of each?

38. Two boys had 45 cents to spend on the Fourth of July, but the elder boy spent 5 cents more than the younger; how many cents did each boy spend?

39. There is a cistern holding 120 gallons, which receives 20 gallons an hour; in how many hours will the cistern be filled, 8 gallons being taken out each hour?

40. Two men traded in company with a capital of $4000, one man putting in 3 times as much as the other. They gained $840; what was each man's share?

41. A master, with his apprentice, can perform in 8 days a piece of work which the master alone can perform in 12 days. In how many days can the apprentice alone do it?

42. A and B can do a piece of work in 12 days, which B and C can do in 9 days, and which B can do alone in 20 days. In how many days could either of the others alone do the work?

43. How long must 50 dollars be kept to be equivalent to the use of 100 dollars 6 months?

SOLUTION. 100 dollars for six months is equivalent to 6 times 100, or 600 dollars, for one month; as many times as $50 are contained in $600, so many months it must be kept. ANS. 12 months.

44. How long must 60 dollars be kept to be equivalent to the use of 80 dollars 9 months?

45. A borrowed of B, at one time, $100, which he kept 5 months, and at another $250, which he kept 2 months; and in payment he lent B $1000; how long must B keep it?

46. If 6 men can build 9 rods of wall in 10 days, how many days will it take 5 men to build 12 rods of the same wall?

SOLUTION. It will take 5 men $\frac{6}{5}$ of 10 days, or 12 days, to build 9 rods of wall; and to build 12 rods it will take them $\frac{12}{9}$ ($\frac{4}{3}$) of 12 days, or 16 days. ANS. 16 days.

47. If 15 men can perform a certain piece of work in 14 days, working 10 hours in the day, how many men will be required to perform an equal amount of work in 10 days, working 7 hours in the day?

48. A vessel sailed from New York with 12 months' provisions for 25 men, but 4 months out she took 15 men from a wreck; how much sooner was she obliged to return to port?

49. If 18 men can dig a trench 24 feet long, 6 feet wide, and 4 feet deep, in 6 days, how many men must be employed to dig a trench 36 feet long, 4 feet wide, and 6 feet deep, in 9 days?

50. A woman bought some apples at 3 for a cent, and as many more at 2 for a cent. She sold them at 5 for 2 cents, and found she had lost 6 cents; how many of each did she buy?

51. A man sold a horse and chaise for 300 dollars; $\frac{1}{3}$ the price of the horse was equal to $\frac{1}{4}$ the price of the chaise; what was the price of each?

52. Two persons work together for the same wages. A saves $\frac{1}{6}$ of his; but B, by spending $5 a month more than A, runs in debt $12 in 6 months; how much does each earn in a month?

53. There is a fish whose head is 12 inches long, his tail is as long as his head and half his body, and his body is as long as his head and tail both; what is the length of the fish?

54. A man spent $\frac{1}{3}$ of his money, and lost $\frac{1}{4}$ of the remainder; he gave a poor woman $\frac{1}{2}$ of what he then had, which left him $12; how much had he at first?

55. What is the time in New York city when it is 5 o'clock in Rome?

56. What part of the principal is the interest at 6 per cent. for 2 years and 6 months? for 3 years and 2 months? for 5 years and 10 months? for 9 years and 2 months?

57. American Independence was declared on the Fourth of July, 1776; how long is it since?

EXPLANATION OF THE SECTIONS IN CONTRACTIONS.

The principles of these contractions can easily be shown to the pupil by multiplying in the usual way, and analyzing fully every step in the process. The product of any two numbers, of two figures each, consists of the product of the tens, plus the product of the tens of the multiplicand by the units of the multiplier, plus the product of the tens of the multiplier by the units of the multiplicand, plus the product of the units. Thus, 16×17 is 272.

The product of the tens (10×10) is 100. The product of the tens of the multiplicand by the units of the multiplier (10×7) is 70, and the product of the tens of the multiplier by the units (10×6) is 60. The product of the units (7×6) is 42. $100 + 70 + 60 + 42 = 272$. The same result is obtained by the method shown in Section XLV.

It may be shown in a similar way how the result is obtained when the sum of the units is 10, and the tens' figures are alike.

The product of 27×23 is 621. 7 times 20 and 3 times 20 are equal to 10 times 20; 20 times 20 and 10 times 20 are equal to 30 times 20. Therefore, instead of multiplying 20 by 7, and by 3, and by 20, we multiply 20 by 30, or 2 tens by

3 tens, and prefix the product to the product of the units, which is 21.

When the sum of the tens is 10 and the units' figures are alike, it is evident that the product of the tens by the units' figure will be as many hundreds as there are units in the units' figure. Thus, 30 times 3 and 70 times 3 are equivalent to 100 times 3, or 3 times 100 = 300.

74 × 34 is 2516. 70 times 30 are 2100; 70 times 4 and 30 times 4 are equivalent to 100 times 4, or 400; 4 times 4 are 16. 2100 + 400 + 16 = 2516.

The principles of the other contractions can be thus readily explained.

For a fuller development of contractions, the pupil is referred to the revised edition of the Theoretical and Practical Arithmetic.